DOCUMENT RESUME

ED 294 763 SE 049 167

TITLE Junior High Mathematics Program of Studies and Curriculum Guide. 1987. Draft.
INSTITUTION Alberta Dept. of Education, Edmonton.
PUB DATE 87
NOTE 348p.
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052) -- Reports - Descriptive (141)

EDRS PRICE MF01/PC14 Plus Postage.
DESCRIPTORS Algebra; *Course Objectives; Curriculum Guides; *Foreign Countries; *Junior High Schools; Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; Number Systems; Problem Solving; Ratios (Mathematics); Secondary Education; Secondary School Curriculum; *Secondary School Mathematics
IDENTIFIERS *Alberta

ABSTRACT

This document includes statements of philosophy and a rationale for the Alberta (Canada) junior high school mathematics program, goals and objectives, a discussion of appropriate content and methods, and a delineation of content using six strands. These include: (1) problem solving; (2) number systems and operations; (3) ratio and proportion; (4) measurement and geometry; (5) data management; and (6) algebra. One volume provides the above information in the form of a program of studies, while the second volume is a curriculum guide. (TW)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED294763

SE 049127

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☒ This document has been reproduced as received from the person or organization originating it

☐ Minor changes have been made to improve reproduction quality

- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

JUNIOR HIGH MATHEMATICS

PROGRAM OF STUDIES

This is an interim program of studies for the junior high mathematics program. This program is scheduled for mandatory implementation in **September 1988**.

MATHEMATICS

A. PROGRAM RATIONALE AND PHILOSOPHY

Mathematics is an important component of education because it enables citizens to lead useful and productive lives and to be adaptive in an ever-changing technological society. The study of mathematics leads to a better understanding and appreciation of the quantitative and geometric nature of the concrete world and to the development of the knowledge, skills and positive attitudes necessary for decision making in personal living. All students should receive a level of mathematics education appropriate to their needs and abilities.

A mathematics program must provide a balance between a knowledge base and the application of that knowledge, especially in new situations and with new technologies. The pervasiveness of calculators and microcomputers and the increasing reliance of the economy on information transfer and processing have changed the ways in which mathematics is used in our society. The result is a substantial (and ongoing) change in emphasis within the familiar mathematical topics such as computational facility, problem solving, measurement and geometry.

The development of positive attitudes toward mathematics and learning is an essential element of a mathematics program in that it nurtures the confidence necessary for taking risks, accepting challenges and making decisions. Positive attitudes are generated by making mathematics meaningful and relevant to students, by selecting activities that are appropriate to students' abilities and by providing opportunities for students to experience success.

Each student must be viewed holistically and as capable of learning. Since self-concept influences learning and achievement, the program should encourage in each student a positive self-concept, and should focus on the growth of each individual. Appropriate and varying organizational and instructional strategies should be implemented to meet the diverse and individual needs of students.

Although junior high school students are at various stages of physical, emotional, and cognitive development, they all require experiences at a concrete level. Extensive experiences with concrete representations of mathematical concepts lead to intuitive understandings of abstractions. Students should be carefully guided from the concrete (model), through the transitional (pictorial representation) and eventually on to the formal (symbolic) level of cognition as mathematics concepts are being developed.

Junior high school students are in a transitional stage of life. Adolescence, characterized by rapid physical growth and the onset of puberty, is a period of uncertainty and great concern about peer relationships. The physical, intellectual, emotional, and social development of the students vary greatly. Supportive comments and guidance, and a genuine expression of concern for students, can help to maintain meaningful communication with students and enhance their learning.

The aim of the Junior High Mathematics Program is to develop an understanding of mathematics concepts by making mathematics relevant and concrete. The emphasis within the program must reflect the reality of the technological age. Appropriate experiences presented in a logical sequence will result in positive attitudes and positive learning outcomes.

B. GOALS AND OBJECTIVES

The goals of the Junior High Mathematics Program are to enable students to:

- solve problems and to grow in their capability to deal with new or different situations.
- use mathematics as a tool in the pursuit of personal goals and aspirations.
- develop a positive self-concept and a positive attitude toward mathematics and lifelong learning.

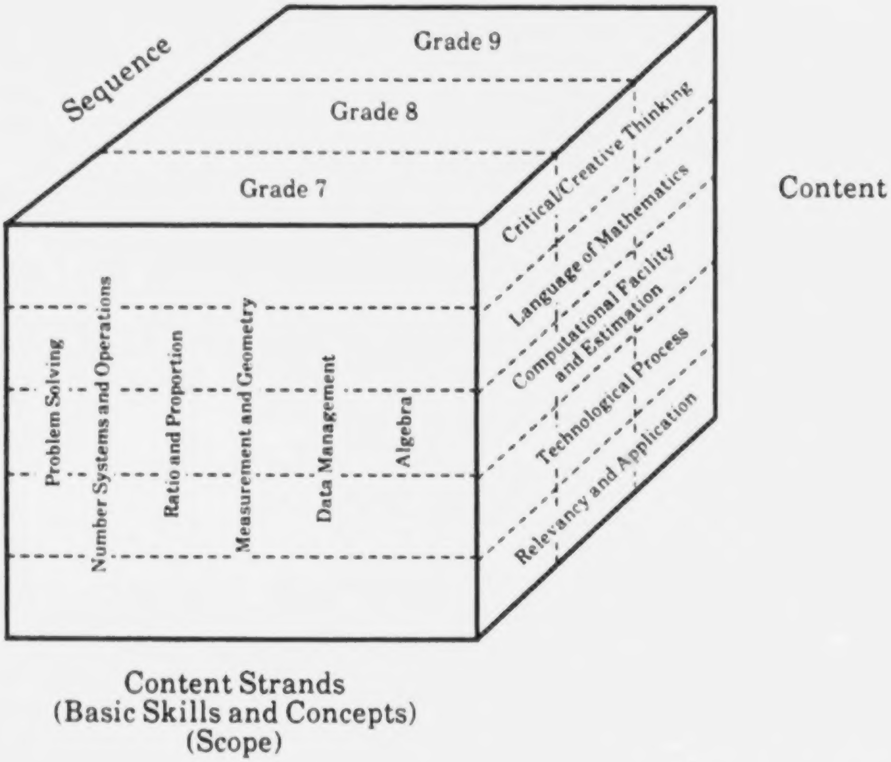
C. CONTENT

STRUCTURE OF THE PROGRAM

The content of the Junior High Mathematics Program is divided into six strands: problem solving; number systems and operations; ratio and proportion; measurement and geometry; data management; and algebra. The content is a consolidation of the skills and concepts developed in the elementary program and forms the basis for the further study of mathematics at the senior high school level. The skills and concepts within these strands are carefully sequenced over three grades taking into account the developmental nature of mathematics and the developmental nature of the learner. All students enrolled in this program should have an opportunity to complete it successfully.

There is an implicit dimension of the mathematics program that transcends the scope and sequence. It cannot be discretely taught as a unit of study nor can it be found in a chapter of a textbook. The context of the program is the element of teaching that creates and fosters positive attitudes, builds appropriate mindsets, and helps the learner interpret and understand the environment in relation to mathematics. Critical and creative thinking, the acquisition of quantitative concepts and skills (number sense), knowledge about and willingness to use technology, knowledge of the language and history of mathematics and the meaningfulness and relevancy of mathematics, must be modelled on a continuous basis and must be integrated into all strands of the program.

JUNIOR HIGH MATHEMATICS PROGRAM DIMENSIONS


Sequence
Grade 9
Grade 8
Grade 7
Content
Critical/Creative Thinking
Language of Mathematics
Computational Facility and Estimation
Technological Process
Relevancy and Application
Problem Solving
Number Systems and Operations
Ratio and Proportion
Measurement and Geometry
Data Management
Algebra
Content Strands
(Basic Skills and Concepts)
(Scope)

The teacher can model and integrate these aspects of the mathematics program through his or her mediation or explanation to students. Understandings are learned, modified and refined over time, eventually building conceptions similar to what the teacher has in mind. The teacher observes students at a task and actively refines their understanding until the desired learning outcome is obtained. Teachers help students interpret these tasks by what they say about them (or by what they leave unsaid). For example, teachers who talk about the perplexing nature of problem solving are likely to impart to students the understanding that perplexity is a normal state in solving problems.

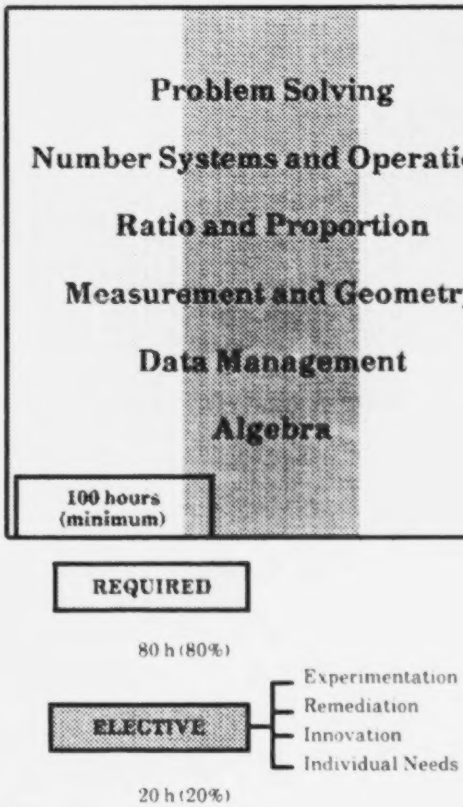
Student understandings and learning outcomes may not always be what teachers intend. For example, when students are always given busy work on computation, the understanding that they may develop is that "getting done" is more important than learning to compute. Students construct understandings about what is important, what to pay attention to and how to behave, from their own experiences and from tasks they encounter within the school experience. New experiences and tasks are combined with old understandings to build new understandings and conceptions.

REQUIRED-ELECTIVE FORMAT

The Junior High Mathematics Program has two components. The required component of the course outlined by the scope of the program describes the basic skills, knowledge and attitudes that all students should be expected to acquire. Of the 100-hour **minimum requirement** for the program, 80% (or 80 hours) shall be spent on this element of the course.

The elective component of the program shall be used to adapt and enhance the required portion of the course to meet the diverse and individual needs and capabilities of individual students. The activities associated with the elective must be integrated throughout the required component and shall be used to remediate and enrich student learning and/or to innovate and experiment with varying instructional and organizational strategies that may enhance student learning. The elective component is not intended to provide acceleration or advanced placement and therefore avoids unnecessary overlap with other courses or courses at a higher level. The maximum time allotment for the elective component shall be 20% of the instructional time.

In cases where the time allotted to the Junior High Mathematics Program exceeds the 100-hour minimum requirement, additional content may be presented to all the students. This content should extend and enhance the understanding of the knowledge, skills and attitudes in the required portion of the program.



SCOPE OF THE PROGRAM

i. Problem Solving

The most important goal of mathematics instruction is the development of students' ability to solve problems. The emphasis on problem solving requires a change in focus from exclusively finding answers to routine word problems to the acquisition and application of many different skills and strategies. Students should be able to apply these strategies to a variety of problem situations where the solutions are unknown and the means to the solution are not immediately evident.

Although problem solving is a legitimate goal in its own right, it should not be viewed as an isolated activity but, rather, as a group of related skills that are a part of a mathematics program. Because of the emphasis it must receive, problem solving appears both as a strand in the program and in an integrated form. The stages of problem solving and a variety of specific skills and strategies are identified and then developed within the strand. The skills, strategies and attitudes associated with problem solving are integrated into the rest of the program and should become part of the teaching philosophy.

ii. Number Systems and Operations

Quantitative thinking and understanding and computational facility are still important goals of mathematics instruction. However, there must be a recognition that there are several ways to compute and today's students must be adept at all the methods. Students must be able to decide which method is most appropriate to the situation at hand and what degree of precision and accuracy is required.

Mental computation, paper-and-pencil operations, estimation and the use of calculators and/or microcomputers are computational strategies that must replace the singular emphasis on paper-and-pencil facility. Paper-and-pencil drills on arithmetic operations with more than three-digit numbers must be de-emphasized. Facility with one-digit number facts must be maintained. Activities that develop number sense and demonstrate the utility of mathematics in problem-solving situations shall increase in emphasis.

Working with numbers and number operations in a real world, problem-solving context gives meaning to numbers and to the operations with them. This is especially true of fractions and decimals. Emphasis shall be placed on the understanding of fractions and decimals as numbers and the comparisons of, and conversions between, fractions and decimals. Drill on operations of fractions with large denominators or multi-place decimals should be de-emphasized.

Mental computation involves finding natural and easy (not formal and algorithmic) strategies for calculations and results in an understanding of number relationships that cannot be replaced by technology. An understanding of the basic properties of number operations shall be developed for the purpose of doing mental calculations.

A heavy emphasis shall be placed on estimating measures and computations (including those that appear in complicated forms). Estimation requires a feel for numbers that goes beyond formal round-off procedures. Students must develop an estimation mindset that includes knowing what an estimate is, accepting its legitimacy, sensing when it is appropriate to estimate, recognizing how precise an estimate should be for a given situation and when a computed answer is sensible.

iii. Ratio and Proportion

Ratio and proportion concepts, although they are an extension of the number systems and operations strand, have been collectively identified as a strand for the purpose of emphasis. The importance and use of equivalent representations in areas such as comparative shopping, scale drawings, model building, map reading, calculating wages, understanding and computing percents, and problem solving, as well as in the study of pure mathematics, cannot be over emphasized. A basic understanding of ratio and proportion must be developed at a concrete level. The applications of ratio and proportion, and percent are numerous and should be made meaningful and relevant to students.

iv. Measurement and Geometry

SI metric measurement concepts and skills need to be consolidated in junior high school. Concrete experiences with making direct comparisons of objects with arbitrary units (e.g., the hand) and with standard units of length, area, volume, capacity and mass (e.g., cm, km^2, m^3, L, g) shall be provided. The need for large and small units of measure and the need to subdivide units into fractional parts should be emphasized. Formulas must be treated as useful tools for finding indirect measurements (e.g., speed) and for finding measurements indirectly (e.g., area). They shall be used after students understand the measure they are to calculate. Excessive memorization of formulas is discouraged.

Geometry is the study of the attributes and properties of various shapes and objects. Attributes to be considered are size and shape of one-, two-, and three-dimensional objects and the transformations of one- and two-dimensional shapes. The measurement of geometric attributes is best done in the context of measurement.

v. Data Management

People are confronted daily with data from which they must make personal and career decisions. Students must cope effectively with the vast amounts of data that they encounter. The importance of statistics, techniques for collecting and interpreting data, making predictions from data and techniques for organizing and displaying data will constitute this strand.

vi. Algebra

Algebra and algebraic thinking are not restricted to courses in the high school. From the time students enter school, they learn about generalizations in the form of symbolism, relations and functions. Open sentences ($\square + 2 = 8$) are used to express basic addition facts; ordered pairs are learned as a part of language development (associating a name with an object); relationships among numbers are learned through counting (less than, equal to, or greater than); and functions which have a unique ordered pair, given the first number, are used in learning basic number facts (e.g., in learning the three-times multiplication table, the set of answers 3, 6, 9... are a function of the counting numbers 1, 2, 3...). Graphs are pictorial representations of the relationship between unique pairs of numbers (e.g., heights of students plotted versus age of students).

THE ROLE OF CALCULATORS AND COMPUTERS

The rapid growth of microtechnology has had an immense impact on mathematics education. Standard computations and manipulations of algebraic symbols, for example, are now incidental applications of hand-held calculators. Mathematics programs must recognize the pervasiveness of technology by de-emphasizing activities that are much more easily replicated by computers, calculators and, in the future, by as yet unknown technologies. Emphasis must be placed on problem solving and on understanding concepts and relationships. Technologies such as computers and calculators must be used to develop concepts, to explore relationships, to explore patterns, to organize and display data, and to eliminate tedious computations.

COURSE OUTLINE

GRADE 7

Problem Solving

1. Demonstrates an understanding of a problem-solving situation.

2. Demonstrates a willingness to find a solution to a problem.

3. Uses a variety of strategies to solve problems.

   The following strategies should be developed throughout the various strands of the program and within the problem-solving framework:

   a. Understanding the problem
   - knows the meaning of all the words in the problem
   - identifies key words
   - draws a diagram
   - classifies information as insufficient or extraneous
   - restates the problem in own words
   - uses concrete manipulatives
   - looks for a pattern
   - considers an alternative interpretation

   b. Developing a plan (choosing a strategy)
   - guesses and checks - improves the guess
   - chooses and sequences mathematical operations
   - acts out or simulates the problem
   - applies a pattern
   - uses a simpler problem

   c. Carrying out the plan
   - applies selected strategies
   - presents ideas clearly
   - documents the process
   - works with care
   - works in a group situation

   d. Looking back
   - determines if the answer is reasonable
   - explains the answer in oral and written form
   - states the solution to the problem
   - restates the problem with the answer
   - considers other possible solutions to the problem
   - looks for other ways to solve the problem
   - discusses solution process with others

Number Systems and Operations

1. Applies and practises problem-solving skills in new situations.

2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.

A. *Whole Numbers*

   1. Maintains previously developed skills with whole numbers (place value, standard and expanded forms, adding, subtracting, multiplying and dividing whole numbers).
   2. Understands properties of number operations and uses properties and relationships to perform mental computations (e.g., associative, commutative, distributive).
   3. Understands that division by zero is undefined.
   4. Writes the value of a power (whole number base and exponent).
   5. Applies the rules for the order of operations to evaluate expressions.
   6. Recognizes prime numbers (limit: primes to 50).
   7. Lists the set of factors for whole numbers up to 200.
   8. Expresses a number as a product of its prime factors.
   9. Uses a calculator or microcomputer to generate a set of multiples of a given number.
   10. Determines whether a number is divisible by 2, 3, 5, 6, 9 or 10.

B. *Decimals*

1. Maintains previously developed skills with decimal numbers (place value, expanded and standard forms, adding, subtracting, multiplying and dividing decimal numbers).
2. Compares and orders decimal numbers.
3. Rounds decimal numbers.

C. *Fractions*

1. Maintains previously developed skills with fractions (concept of a fraction, need for fractional numbers, equivalent fractions, basic fractions) at a concrete level.
2. Identifies mixed numbers and improper fractions and converts from one to the other.
3. Orders fractional numbers.
4. Uses concrete manipulatives to demonstrate the addition and subtraction of fractions with and without common denominators.
5. Writes number sentences to describe the addition and subtraction of fractions.
6. Uses concrete manipulatives to demonstrate the multiplication and division of proper fractions.
7. Writes number sentences to describe the multiplication and division of fractions.

D. *Integers*

1. Maintains previously developed skills with integers (concept of integers, need for integers, ordering of integers).
2. Uses concrete manipulatives to demonstrate the addition of integers.
3. Writes number sentences to describe addition of integers.

Ratio and Proportion

1. Applies and practises problem-solving skills in new situations.

2. Maintains previously developed skills (identifies ratios as ordered pairs of numbers related to concrete situations; uses whole number constants to generate equivalent ratios).

3. Uses concrete manipulatives to construct ratios in the following forms:

$$a{:}b,\ a \text{ as to } b, \text{ and } \frac{a}{b}$$

4. Verifies the equivalence of two ratios using common multiples or factors:

$$\text{e.g., } \begin{array}{ccc} \underline{14} & (\div 2) \rightarrow & \underline{7} \\ 6 & (\div 2) \rightarrow & 3 \end{array}$$

5. Finds a missing element of a proportion using a common multiple of the elements:

$$\text{e.g., } \begin{array}{ccc} \underline{3} & (\times 3) \rightarrow & \underline{x} \\ 4 & (\times 3) \rightarrow & 12 \end{array}$$

6. Identifies percent as a ratio:

$$\text{e.g., } \left(p{:}100 \text{ or } \frac{p}{100} \right)$$

7. Expresses ratios as percents and decimals and vice versa (limit: ratios in the form a:b, where b=2, 4, 5, 10, 20, 25, 50):

$$\text{e.g., } \frac{3}{4} \rightleftharpoons \frac{75}{100} \rightleftharpoons 75\%$$

8. Finds the percent of a number:

e.g., 15% of 25

9. Expresses one number as a percent of another number:

e.g., 12 is what percent of 16?

$$\text{or } \frac{12}{16} = \underline{\qquad}\%$$

Measurement and Geometry

1. Applies and practises problem-solving skills in new situations.
2. Maintains previously developed skills (concepts of linear, perimeter, area, volume, capacity and mass measures in concrete, pictorial and formal forms; determines perimeter and area of right triangles and rectangles, and volumes of rectangular solids with and without formulas; uses protractor to determine the measure of an angle; transformational geometry).
3. Expresses equivalent measures of SI units (linear).
4. Understands and uses the terms similar and congruent with respect to geometric figures.
5. Understands and uses the term symmetry with respect to geometric shapes.
6. Constructs geometric designs using tools such as a computer, compass, straightedge, ruler or mira.

Data Management

1. Applies and practises problem-solving skills in new situations.
2. Demonstrates a knowledge and understanding of the use and purposes of statistics as it affects daily living.
3. Collects and records data (tally sheets and frequency tables).
4. Understands and uses the term average (mean) as related to practical situations (e.g., test marks).
5. Interprets data from pictographs, bar graphs, line graphs and circle graphs.
6. Understands when and how to represent data in the form of pictographs, bar graphs, line graphs and circle graphs.

Algebra

1. Applies and practises problem-solving skills in new situations.
2. Understands and uses the term variable and uses variables to describe a concrete situation (e.g., number of jelly beans in a jar).
3. Uses variables to write mathematical expressions to represent practical situations (e.g., age of the students in the class in three years will be x + 3 years).
4. Evaluates expressions for given values of the variable (limit: whole numbers, positive rationals).
5. Uses variables to write mathematical sentences to represent practical situations (e.g., people in a classroom = boys + girls + teachers or p = b + g + t).
6. Uses concrete manipulatives to demonstrate the concept of "equals" (i.e., equality).
7. Uses estimation, and guess and test procedures to solve equations of the form:

$$x+a=b,\ ax=b,\ ax+b=c, \text{ and } \frac{x}{a}=\frac{b}{c}$$

8. Verifies solutions to equations by substitution.
9. Given ordered pairs, plot points on a coordinate plane.

Problem Solving

1. Demonstrates an understanding of a problem-solving situation.

2. Demonstrates a willingness to find a solution to a problem.

3. Uses a variety of strategies to solve problems. Previously developed strategies are used.

   The following strategies should be developed throughout the various strands of the program and within the problem-solving framework:

   a. **Understanding the problem**
      - interprets pictures, charts and graphs
      - asks relevant questions

   b. **Developing a plan (choosing a strategy)**
      - collects and organizes information (charts and graphs)
      - makes diagrams and models
      - experiments through the use of manipulatives
      - breaks the problem into smaller parts
      - works backwards

   c. **Carrying out the plan**
      - applies selected strategies
      - presents ideas clearly
      - documents the process
      - works with care
      - works in a group situation

   d. **Looking back**
      - makes and solves similar problems

Number Systems and Operations

1. Applies and practises problem-solving skills in new situations.

2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.

A. *Whole Numbers*

1. Maintains previously developed skills with whole numbers (operations, order of operations, evaluation of expressions, prime numbers, factorization, divisibility).

2. Finds the greatest common factor of a given set of numbers.

3. Finds the lowest common multiple of a given set of numbers.

4. Understands and uses the terms exponent, base, power, squared and cubed and the n^{th} power of a number.

5. Demonstrates the need for scientific notation.

6. Writes numbers in scientific notation, and scientific notation numbers in standard form (limit: positive exponents).

B. *Integers*

1. Maintains previously developed skills with integers (need for integers, concept of integers, ordering of integers, demonstrates addition of integers with manipulatives).

2. States the additive inverse of any integer.

3. Uses concrete manipulatives to demonstrate the subtraction, multiplication and division of integers.

4. Performs the operations of addition, subtraction, multiplication and division with integers using paper-and-pencil algorithms, estimation, mental computation and a calculator.

C. *Rational Numbers*

1. Maintains previously developed skills with decimal numbers (place value, operations, ordering, rounding, order of operations).

2. Maintains previously developed skills with fractional numbers (concept of a fraction, equivalent fraction, basic fraction, mixed numbers, improper fraction, ordering fractions, concrete operations with fractions, order of operations).

3. Writes the multiplicative inverse (reciprocal) of a fraction, whole number or integer.

4. Performs the operations of addition, subtraction, multiplication and division with fractions (limit: positive rationals).

5. Demonstrates the need for rational numbers (e.g., $-3 \div 2 = ?$ No answer is possible without a set of rational numbers).

6. Recognizes rational numbers as all numbers that can be written in the form:

   $\frac{a}{b}$ where $b \neq 0$.

7. Compares and orders rational numbers using $<$, $>$ or $=$.

8. Uses a number line to demonstrate the relationship between whole numbers, integers, fractions and rationals.

Ratio and Proportion

1. Applies and practises problem-solving skills in new situations.

2. Maintains previously developed skills (understands and constructs ratios, equivalent ratios; finds missing element of a proportion, percent as a ratio, percents as decimals, percents of numbers; and expresses one number as a percent of another).

3. Gives examples of ratios involving situations where the equivalent percent is greater than 100.

4. Converts mixed numbers to percents and vice versa.

5. Given the percent determines the missing value in applications such as discounts, increases, decreases, or sales tax.

6. Understands and writes rates as the comparison of two numbers with different units (e.g., 15 km/2h or 3 items/$1).

7. Writes proportions involving rates.

8. Finds the missing element in a proportion involving rates.

Measurement and Geometry

1. Applies and practises problem-solving skills in new situations.

2. Maintains previously developed skills (linear, area, volume, capacity and mass units of measure; uses geometric tools to measure line segments and angles and to construct geometric designs; transformational geometry).

3. Understands and uses the terms perpendicular and parallel lines.

4. Draws or sketches various polygons using tools such as a computer, compass, straightedge, ruler, protractor.

5. Identifies and classifies polygons according to the number of sides (limit: decagon).

6. Investigates triangles by examining attributes such as measure of angles, measure of sides and lines of symmetry.

7. Investigates quadrilaterals by examining attributes such as measure of sides, measure of angles, lines of symmetry and diagonals.

8. Adds, subtracts, multiplies and divides using SI units of measure.

9. Understands and uses formulas as indirect measures of the perimeter of polygons (includes regular polygons).

10. Understands and uses formulas as indirect measures of the area of polygons (triangles, all parallelograms and trapezoids).

11. Performs an experiment to determine the value of π and understands π as a ratio of the circumference of a circle divided by its diameter.

$$(i.e.,\ \pi = \frac{C}{d})$$

12. Understands and uses the formula $C = \pi d$ as an indirect measure of the circumference of a circle.

13. Uses the formula $A = \pi r^2$ to indirectly determine the area of a circle given its radius or diameter.

14. Draws or sketches a right rectangular prism.

15. Understands and uses a formula as an indirect strategy for determining the volume of a right rectangular prism or a cube.

Data Management

1. Applies and practises problem-solving skills in new situations.

2. Maintains previously developed skills (understands the purpose of statistics; interprets data from tables and graphs; draws graphs).

3. Understands and uses the terms bias, sample and population.

4. Distinguishes between a survey and a census, understands when each is used and potential biases that may occur (survey).

5. Recognizes the use and misuse of statistics in society (news reporting, census, polls, etc.).

Algebra

1. Applies and practises problem-solving skills in new situations.

2. Maintains previously developed skills (variable, evaluation of expressions, concept of equality, plots on a coordinate plane).

3. Identifies and combines like terms.

4. Uses formal procedures to solve equations of the form:

$$x + a = b,\ ax = b,\ ax + b = c, \text{ and } \frac{x}{a} = \frac{b}{c}$$

(limit: positive rational numbers and integers).

5. Verifies solutions to the equations.

6. Uses substitution and equation-solving techniques to find a missing element of a formula:

e.g., If $p = 2$ and $q = 0.5$ find c in $p = \frac{c}{q}$

7. Generates a set of ordered pairs in a linear relation.

8. Given a linear relation, constructs a table of values and a graph for that relation.

GRADE 9

Problem Solving

1. Demonstrates an understanding of a problem-solving situation.

2. Demonstrates a willingness to find a solution to a problem.

3. Uses a variety of strategies to solve problems. Previously developed strategies are used.

   The following strategies should be developed throughout the various strands of the program and within the problem-solving framework:

   a. **Understanding the problem**
      - considers alternative interpretations
      - makes assumptions

   b. **Developing a plan (choosing a strategy)**
      - formulates an equation
      - uses logic or reason
      - constructs flow charts
      - develops a symbol or code system
      - recognizes limits and eliminates possibilities

   c. **Carrying out the plan**
      - applies selected strategies
      - presents ideas clearly
      - documents the process
      - works with care
      - works in a group situation

   d. **Looking back**
      - generalizes solutions
      - creates and writes routine and non-routine problems

Number Systems and Operations

1. Applies and practises problem-solving skills in new situations.

2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.

3. Maintains previously developed skills with whole numbers, integers, decimals and fractions (operations, ordering, relationships among systems, need for rational numbers, order of operations).

4. Performs the operations of addition, subtraction, multiplication and division with rational numbers.

5. Applies the rules for order of operations to evaluate expressions involving rational numbers in any of their forms.

6. Converts rational numbers from $\frac{\text{"a"}}{\text{b}}$ form to decimal form (limit: $b < 10$ or b is a power of 10).

7. Converts rational numbers from decimal form to $\frac{\text{"a"}}{\text{b}}$ form (limit: terminating decimals).

8. Computes the square root of whole numbers using estimation and a calculator.

9. Demonstrates the relationship among whole numbers, integers and rational numbers.

10. Understands and uses the following properties (limit: numerical bases):
    - $a^x \times a^y = a^{x+y}$
    - $a^x \div a^y = a^{x-y}$
    - $(a^x)^y = a^{xy}$
    - $a^1 = a$
    - $a^\circ = 1, a \neq 0$
    - $a^{-x} = \frac{1}{a^x}$ (limit: $a = 10$)

11. Writes large and small numbers in scientific notation:

    (e.g., $0.00008 = 8 \times 10^{-5}$)

Ratio and Proportion

1. Applies and practises problem-solving skills in new situations.

2. Maintains previously developed skills (understands and constructs ratios, rates and proportions; finds the missing element of a proportion; writes ratios as percents; converts fractions and decimals to percents and percents to fraction and decimal forms; finds missing values in commission, sales tax, and discount situations).

3. Converts fractional percents to fraction and decimal forms:

$$\text{e.g., } 12\tfrac{1}{2}\% = \frac{1}{8} = 0.125$$

4. Finds any one of the missing elements (value or percent) in applications such as simple interest, commission, sales tax, discount, profit and loss, and percent increase and decrease situations.

5. Interprets maps and scale drawings.

6. Uses a scale to construct drawings, maps or pictures.

7. Applies ratio and proportion in practical situations (e.g., uses shadows to find the height of a pole or building; comparative shopping; building a model, computing a test or report card mark based on weighted averages).

Measurement and Geometry

1. Applies and practises problem-solving skills in new situations.

2. Maintains previously developed skills (linear, area, volume, capacity, and mass units of measure; classification of polygons; perimeter and area of polygons and the circle; volume of a right rectangular prism and cube).

3. Uses concrete manipulatives to determine the sum of the angles in a triangle (180°).

4. Determines the sum of the interior angles in polygons.

5. Uses concrete manipulatives to develop the Pythagorean relationship in right triangles.

6. Applies the Pythagorean relationship to practical situations.

7. Constructs regular polygons using tools such as a computer, ruler, protractor and/or compass.

8. Understands and uses a strategy to determine the area of a regular polygon.

9. Identifies pairs of angles: (supplementary, complementary, adjacent and opposite).

10. Uses a compass and a straightedge to construct:
    - a congruent segment
    - a congruent angle
    - a perpendicular bisector of a segment
    - a bisector of an angle
    - a perpendicular to a line
    - angles of 90°, 45°, 60° and 30°.

11. Given nets, constructs right prisms.

12. Classifies right prisms and cylinders.

13. Understands and uses a strategy for finding the surface area of any right prism or cylinder.

14. Understands and uses a strategy for finding the volume of any right prism or cylinder.

Data Management

1. Applies and practises problem-solving skills in new situations.

2. Maintains previously developed skills (understands purpose, use and misuse of statistics; biases in surveys; represents data in the form of pictographs, bar graphs, line graphs, circle graphs).

3. Analyzes and interprets arguments or conclusions based on statistical information.

4. In data from meaningful situations (e.g., test marks), understands and uses the terms mean, median, mode and range.

5. Distinguishes between a percent and a percentile.

6. Conducts a survey or poll using correct sampling techniques and reports results using an appropriate table, chart and/or graph.

7. Understands and uses the term probability.

8. Expresses the probability of the occurrence of an event from a practical situation or a simple experiment or simulation (e.g., pulling a particular coloured marble out of a bag or socks out of a drawer).

Algebra

1. Applies and practises problem-solving skills in new situations.

2. Maintains previously developed skills (variables; like terms, evaluation of expressions; solving equations; generating and plotting ordered pairs from a given relation).

3. Uses formal procedures to solve equations (using all forms of rationals) of the form

$$x + a = b,\ ax = b,\ ax + b = c, \text{ and } \frac{x}{a} = \frac{b}{c}$$

$$ax + b = cx, a(x + b) = c$$

$$\text{and } ax + b = cx + d$$

4. Verifies solutions to equations.

5. Manipulates a given formula to change the subject of the formula:

e.g., given $x = \frac{f}{w}$ then $w = \frac{f}{x}$

6. Finds a missing element of a formula through manipulation.

7. Solves inequalities of the form $x + a \geq b$ and $cx \leq d$ (c is positive; direction of inequalities may vary).

8. Verifies solutions to inequalities.

9. Graphs solutions to inequalities on a number line.

10. Given a set of ordered pairs or a table of values, writes the function that determines the relation (limit: linear relations).

JUNIOR HIGH MATHEMATICS

CURRICULUM GUIDE

1987

DRAFT

Alberta
EDUCATION

TABLE OF CONTENTS

Acknowledgements ii

GRADE 7 MATHEMATICS

Problem Solving 4
Number Systems and Operations 12
Ratio and Proportion 42
Measurement and Geometry 48
Data Management 56
Algebra 62

GRADE 8 MATHEMATICS

Problem Solving 74
Number Systems and Operations 82
Ratio and Proportion 102
Measurement and Geometry 108
Data Management 120
Algebra 126

GRADE 9 MATHEMATICS

Problem Solving 134
Number Systems and Operations 138
Ratio and Proportion 150
Measurement and Geometry 154
Data Management 164
Algebra 170
Algebra–Extended Content 176

ACKNOWLEDGEMENTS

This curriculum guide draft has been prepared with guidance and input provided by a number of individuals. Their suggestions and assistance has proved to be valuable in the development of this document.

JUNIOR HIGH MATHEMATICS ADVISORY COMMITTEE

Mr. Pat McLaughlin (Chairman), Alberta Education
Ms. Linda Cox, Alberta Correspondence School
Mr. Don Darling, Edmonton School District #7
Dr. Tom Kieren, University of Alberta
Mr. Rene Mathieu, Alberta Education
Mr. Garry Canadine, Lethbridge School District #51
Ms. Teresa Scott, Grande Prairie School District #2357
Mr. Bruce Stonell, Stettler School District #1475
Mr. Glen Wilson, County of Minburn #27

JUNIOR HIGH MATHEMATICS DEVELOPMENT COMMITTEE (Ad Hoc)

Mr. John Chapman, High Prairie School Division #48
Ms. Jean Crawford, Calgary Roman Catholic Separate School District #1
Ms. Myra Hood, Calgary School District #19
Mr. Ken Hutchings, County of Strathcona #20
Mr. Dick Kopan, Calgary School District #19
Mr. Peter Miller, County of Flagstaff #29
Ms. Katie Pallos-Haden, County of Parkland #31
Mr. Leo Sam, County of Smoky Lake #13
Ms. Jan Sparling, Teacher, St. Albert
Ms. Susan Turner, Grande Prairie School District #2357

ADMINISTRATION AND PRODUCTION

This curriculum document has been prepared within the Junior High Unit, Curriculum Branch, Alberta Education.

Jim Brackenbury, Associate Director
Barbara Esdale, Learning Resources Officer
Alexandra Hildebrandt, Learning Resources Officer
Merv Lastiwka, Program Manager
Valerie Perry, Curricular Assistant

Special recognition must go to the following staff members who helped to produce this document:

Kim Blevins, Copy Editor
Lisa McCardle, Editor

Jan Cruickshank, Word Processing
Lin Gray, Word Processing
Denise Hugman, Word Processing
Cheryl Stoochnoff, Word Processing
Janice Yarmuch, Word Processing
Esther Yong, Word Processing

GRADE 7
MATHEMATICS

PLEASE NOTE

The following resources have received basic status and are available from the Learning Resources Distributing Centre:

Student texts for JOURNEYS IN MATH 7, 8
Student texts for HOLTMATH 7, 8, 9

Any other titles mentioned in this guide have not received formal status approval. They are presently under review and cannot, at this time, be obtained through the Learning Resources Distributing Centre.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Demonstrates an understanding of a problem-solving situation.	Problem solving should not be viewed as an isolated activity but rather as a process that is to be an integral part of the teaching philosophy to be used in the development of the other strands. The framework for problem solving should be introduced at the beginning of the year (suggested time – 3 to 5 periods). Brainstorm for a definition and examples. The following ideas should evolve about a problem: a) it has no readily apparent solution or the means to the solution is not immediately evident b) it can cause a person to be temporarily perplexed c) it may have no answer, one answer, or more than one answer	Teachers must recognize that problem-solving skills are essential for all students and that being perplexed when first encountering a problem is normal. Problems presented to students should be challenging yet solutions must be attainable to insure that students experience success. It is very important for teachers to realize individual student differences in learning; therefore the growth expectation should also vary. Individual needs can often be met by changing the conditions of a problem to make it simpler.	a) The use of calculators in problem solving must be encouraged so that time spent on tedious calculations is decreased and feedback on strategies is faster. Numbers from realistic and relevant situations are less imposing if calculators are used. b) Group work should often be used in problem solving. A student in a group deals with ideas and questions from other members of the group, and this may help each student to progress in developing problem-solving strategies.

Note: (E) = Enrichment
(R) = Remediation

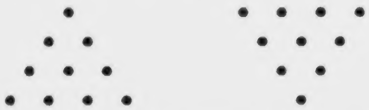
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1 Demonstrates an understanding of a problem-solving situation. (cont'd)	d) it can be of a practical, everyday, personal or social nature as well as of a mathematical nature. (See Journeys in Math 7 TRM, p. 30.)	Manipulatives can also be used to meet individual needs. e.g., Students are given a pile of 21 markers. Two players are involved and take turns removing one, two or three markers. The winner is the player who removes the last marker. The purpose of the game is to develop a strategy to always win. As students continue to work on this they should become more interested in finding a strategy rather than winning. For students who have difficulty with this, decrease the number of markers used or only move one or two markers. Demonstrate how the markers can be grouped and ask students critical questions such as the importance of moving first, and other strategic moves. The game can also be made into a more difficult version to challenge higher ability students. Use two piles and change the rules. Students can take one marker from each pile or one marker from only one pile.	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Demonstrates a willingness to find a solution to a problem.	In order to develop the students' willingness to find solutions, the teacher should: a) create a positive classroom atmosphere that allows students to foster their own ideas and approaches in problem solving b) be supportive and encourage risk taking in finding solutions c) encourage students to use creative approaches d) be willing to accept unconventional solutions, more than one solution, or no solution (where appropriate) e) challenge students to think critically and justify strategies and solutions f) be enthusiastic and capable of recognizing the students' willingness and perseverance to solve problems g) provide appropriate questions for students h) present problem situations that enable students to gain problem-solving experience that is transferable to other subject areas and everyday life.	Students who experience difficulty with the complex strategies may find it necessary to use a more concrete approach for a longer period of time and may require more teacher guidance. e.g., A store owner buys candies in bulk bags containing 80 candies each. He re-packages the candies for sale in smaller bags of 12. How many candies are left over when one bulk bag of 80 candies is re-packaged?	Computers may be used to assist in teaching problem solving. Various programs and simulations require the use of particular or various strategies (e.g., Houghton Mifflin MAC, MECC and Sunburst Communications Software). The use of relevant and realistic problems (from sources such as newspapers and magazines) is encouraged because this will increase the interest of students. Students may also be able to contribute their own ideas of problems to solve.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Demonstrates a willingness to find a solution to a problem. (cont'd)		Use of a concrete example will help students who experience difficulty with the operation of division. Concrete approaches should be encouraged as long as it is necessary for the student. A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider the possibilities such as: a) other strategies and solutions b) "what if?" (change an element of the problem) c) generalization of rules to other situations. e.g., Using the above candy problem, ask: "How many bulk bags of 80 candies each would the store owner need to re-package so that no candies are left over?"	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Uses a variety of strategies to solve problems. The following strategies should be developed throughout the various strands of the program and within the problem-solving framework: a) Understanding the problem • knows the meaning of all the words in the problem • identifies key words • draws a diagram • classifies information as insufficient or extraneous • restates the problem in own words	To introduce the strategies of problem solving, an approach such as the following may be used: Choose three non-related but similar problems that can be solved focussing on a strategy (consider that any problem usually requires the application of more than one strategy) such as the strategy of acting out or simulating the problem. The first problem can be a teacher demonstration, the second can be a student trial with teacher guidance, and the third can be student practice. The strategy of acting or simulating the problem can be developed within the problem-solving framework [Understanding the problem, Developing a plan, Carrying out the plan, Looking back] as follows: If six people were in a room and each one shook hands with every other person, how many handshakes were there? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 42, #29.)		

PROBLEM SOLVING **Grade 7**

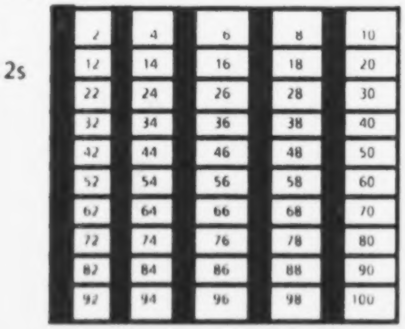
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
a) Understanding the problem (cont'd) • uses concrete manipulatives • looks for a pattern • considers an alternative interpretation b) Developing a plan (choosing a strategy) • guesses and checks - improves the guess • chooses and sequences mathematical operations • acts out or simulates the problem • applies a pattern • uses a simpler problem	Under the guidance of the teacher, the students may now investigate a similar but non-related problem in a group situation by using the strategy of acting out or simulating. One such problem may be: In how many ways can a committee of two be selected from five people? A practice problem involving an acting out (or using manipulatives) strategy may be as follows: Move three coins on the figure on the left to make it look like the figure on the right. (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 19, #7.7.)  The actual use of a manipulative could be very effective for lower ability students. Evaluation The evaluation of problem solving requires more than grading the solutions to mathematical problems. Continual observation and questioning of students while they are solving problems is essential.		

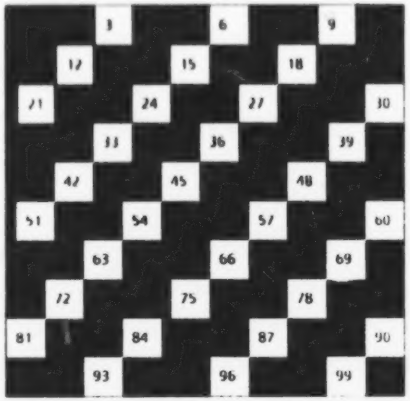
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
c) Carrying out the plan • applies selected strategies • presents ideas clearly • documents the process • works with care • works in a group situation d) Looking back • determines if the answer is reasonable • explains the answer in oral and written form • states the solution to the problem • restates the problem with the answer • considers other possible solutions to the problem • looks for other ways to solve the problem • discusses solution process with others.	In assessing a student's problem-solving skills, the teacher should consider: a) willingness to attempt problems b) use of a systematic approach c) selection of appropriate strategies d) efficiency in selection of appropriate strategies e) logical justification of strategies and solutions f) perseverance g) growth in confidence in problem-solving ability h) transfer of problem-solving skills to situations other than mathematics. Evaluation techniques and instruments for problem solving are found in the Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, pp. 7, 8, 52-56.	Problem solving is integrated as an essential part of each strand. Enrichment/Remediation and use of technological devices is therefore outlined in each strand.	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.	The intent is to develop non-routine problem-solving skills in every student. Each student should understand the concepts of the problem-solving framework and develop multiple strategies. Specific problem-solving techniques and skills will be developed with each objective.		Students must be taught how to use calculator features after understanding of a concept is developed. Memory keys (order of operations) and percent keys operate on fundamental mathematics concepts. Students should be encouraged to discover these concepts. In addition, students must be taught how to interpret results. Knowing how to find a remainder or the repeating period of a rational number are implicit learning outcomes.
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.	An equal emphasis should be placed on the various strategies for computing. Single-digit basic facts should be drilled on a regular basis through activities such as timed challenges or games. Paper-and-pencil strategies should be used to develop an understanding of sub-concepts such as re-grouping, borrowing or place value. Long and tiresome paper-and-pencil drill is discouraged. Estimation should be done on a daily basis. Recognition of appropriate situations for estimates, determining how precise an estimate should be for a given situation, and knowing when a computed answer is possible, are among skills to be emphasized.		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations. (cont'd)	Mental computation involves using natural and easy strategies to compute exact answers. Strategies should be identified and shared as they evolve. Calculators should be used to develop understanding to investigate patterns, and to perform tedious computations that do not enhance understanding.		

<table>
<tr><th>Objective</th><th>Clarification or Example</th><th>Elective Suggestions</th><th>Integration of Problem Solving and Technology</th></tr>
<tr>
<td>A. Whole Numbers
1. Maintains previously developed skills with whole numbers (place value, standard and expanded forms, adding, subtracting, multiplying and dividing whole numbers)</td>
<td>Use a diagnostic tool to determine student proficiency with each of the operations.
Facility with basic facts through activities such as:
(i) "Mad Minute"
This activity is designed to maintain basic skills in a timed situation. Encourage students to complete as many questions as quickly and accurately as possible.
(ii) Drill using computer software Holt Disks 3 and 5, MAC Courseware.
(iii) Engage students in frequent estimation activities, e.g., place blocks/rods on overhead. Have students estimate the number. Students demonstrate their answers with cardboard tiles (0-9). Narrow the range of possible answers by giving clues (odd #, perfect square, palindrome, etc.).
e.g., Using blocks/rods on the overhead to show the number 625. Have students estimate.
Students demonstrate the number with cardboard tiles. Continue giving clues until students display the correct answer.</td>
<td>(R) To reinforce place value encourage the use of manipulatives. Each student should have his/her own material to handle. Make sure that proportional aids are used (these are aids that have a size difference).
e.g., These aids work effectively on the proportional level:
<table>
<tr><th>Exchange this</th><th>for this</th></tr>
<tr><td>10 beans</td><td>1 bean stick</td></tr>
<tr><td>10 white Cuisenaire rods</td><td>1 orange Cuisenaire rod</td></tr>
<tr><td>10 single beads</td><td>1 string of 10 beads</td></tr>
<tr><td>10 squares cut from graph paper</td><td>1 10-square strip of graph paper</td></tr>
</table>
Work slowly with the aid. Once students understand the concept you can try using changeable aids. At this point you can move on to the concept at a more abstract level (adapted from Arithmetic Teacher, Vol. 32, #1, September 1984, p. 21).</td>
<td></td>
</tr>
</table>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 1. Maintains previously developed skills with whole numbers (place value, standard and expanded forms, adding, subtracting, multiplying and dividing whole numbers). (cont'd)		(E) Patterns in the Multiplication Tables. Display a 10 x 10 hundred chart with stencils cut from tagboard which allow only one set of multiples to be displayed. Ask students what they see (Fig. 1). Continue doing the same activity with 3s, 4s, 5s, 6s, 7s, 8s, 9s. To allow all students to participate, have them begin quietly and independently. Have them write notes to themselves and then share ideas. Have students complete a 3s table. Discuss the pattern that emerges. Ask students what happens when 2s and 3s are combined? (6s) Fig. 1. Multiples  2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78 80 82 84 86 88 90 92 94 96 98 100	Guess and check strategy can be useful when reinforcing skills/operations with whole numbers. e.g., Karen sold $50 worth of tickets. Student tickets were $2 and adult tickets were $3. If she sold 10 student tickets, how many adult tickets did she sell? 1) Student guess. 2) Ask questions for understanding of the problem: What are you finding? Price of tickets? Number of tickets sold? etc. 3) Ask questions for solving (hints). 4) Follow up – discuss/focus on the strategy students used – the solution and alternative solutions.

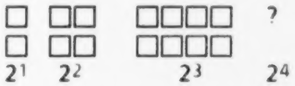
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 1. Maintains previously developed skills with whole numbers (place value, standard and expanded forms, adding, subtracting, multiplying and dividing whole numbers). (cont'd)		3s  3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60 63 66 69 72 75 78 81 84 87 90 93 96 99 (Arithmetic Teacher, Vol. 32, #7, March 1985, pp. 36-37.)	

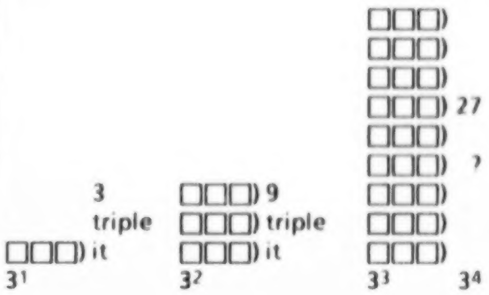
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 2. Understands properties of number operations and uses properties and relationships to perform mental computations (e.g., associative, commutative, distributive).	The intention here is to develop student understanding of the properties through their indirect use. Encourage students to do many "mental exact" computations and then explain the properties. e.g., 1. 23 + 28 + 7 + 2 = 2. 16 + 9 + 4 = 3. 2 × 18 × 10 = 4. 16 × 12 = 10(12) + 6(12) = 5. add mentally from left to right 236 587 2 + 5 = 3 + 8 = 6 + 7 = It is important that students understand the properties and relationships rather than define them. Students can demonstrate their understanding by their ability to verbalize.	(R) Timed drill activities on mental computations can be used to increase student skills.	

47

48

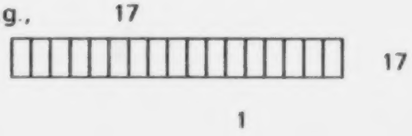
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 3. Understands that division by zero is undefined.	Pattern Using a calculator or computer direct students to divide a given number by a set of numbers that become smaller (approach 0). e.g., 10, 5, 2, 1, $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{100}$ Ask students to predict what would happen if they divided by 0. Check on a calculator. Does an "Error" message appear? Why?	(R) What does division mean? (Repeated subtraction.) Use concrete manipulatives to count how many times, for example, a group of 2 can be subtracted from 6. Then ask "how many times can 0 blocks be removed from 6?" Discussion should lead students to the conclusion that the question itself is meaningless.	

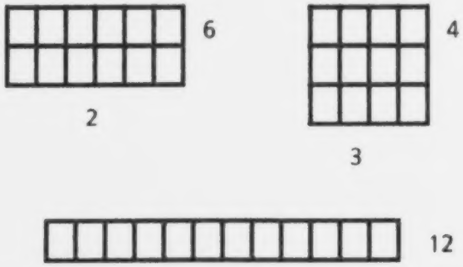
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 4. Writes the value of a power (whole number base and exponent).	Use blocks to demonstrate/investigate patterns. Teachers may wish to use alternative concrete manipulatives such as sticks, beans, coloured squares or grid paper. $2^1 =$ 2 (blocks) $2^2 =$ 4 (blocks) $2^3 =$ 8 (blocks) $2^4 =$ 16 (blocks) etc.  Develop the concept further looking at the patterns with other numerical bases. Explain to students that each result is doubling and therefore increasing the exponent by 1. By investigating patterns with 3s and 4s, students should realize that 3 or 4 is the factor and the number of times it has increased is the exponent.	(R) Extended use of blocks. (E) Reverse the procedure. Look at a variety of patterns leading to an understanding of $x^0 = 1$ at a concrete level. e.g., 2^4 2^3 2^2 2^1 2^0 16 8 4 2 ? 3^3 3^2 3^1 3^0 27 9 3 ?	Using calculators and computers develop the investigation further. (Arithmetic Teacher, Vol. 34, #6, February 1987, p. 36.) Write the values for the powers. e.g., $7^1, 7^2, 7^3, 7^4, 7^5, 7^6$ 7 , , , , , , Describe the pattern. The following activity encourages the development of guess and check. Guess the number that goes in the box □. Check with calculator; continue until the correct number is found $\square^3 = 12167$ $\square^3 = 704969$ $\square^4 = 1419857$ $\square^4 = 16807$ Challenge students to find the number in 3 tries? 4? 5?

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 4. Writes the value of a power (whole number base and exponent) (cont'd)	e.g.,  3 triple it 9 triple it 27 ? 3¹ 3² 3³ 3⁴ Students can continue developing the concept to a factor of 10. They should be able to extend their understanding to include larger numerical factors.		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 5. Applies the rules for the order of operations to evaluate expressions.	The intent is to pose a problem involving the order of operations. Students should use calculators and computers to find a solution for 67 − 8 × 3. As two solutions emerge, focus discussion on order of operations. The following activity will reinforce this skill: (73 □ 26) □ 23 = 2277 (62 □ 21) □ 236 = 1066 1776 = (882 □ 49) □ 1758 215 □ 896 □ 788 □ 412 = 735 The use of the calculator can help determine the missing sign. Encourage students to use mental estimation to help them.	(E) Calculator. Challenge students to develop a method which uses the memory function to solve problems using the order of operations e.g., 67 - 8x3⇔MC 8x3 = M + 67 - MR = 43	

NUMBER SYSTEMS AND OPERATIONS Grade 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A Whole Numbers 6 Recognizes prime numbers (limit: primes to 50).	Students should make all possible rectangles using a given prime number. This concept is fully developed for factors and can be extended to prime numbers. e.g.,  Students will discover that only one possible example exists.	(R) Reinforcement. Continue use of manipulatives: e.g., Sieve of Eratosthenes. A method of making a list of prime numbers was devised by an ancient Greek scholar, Eratosthenes, in the third century B.C. The method consists of taking a 100 chart and circling the first five prime numbers, 2, 3, 5, 7, and 11. Then all numbers divisible by 2, 3, 5, 7, and 11 are crossed out. The prime numbers will not be crossed out. 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 (R) Prime Number Game. Give students a list of the numbers from 1-50. Player 1 chooses a number (e.g., 15). This is crossed off the list. Player 2 strikes out all the factors of 15, (1, 3, 5) and adds them together to score 9 points. Player 1 again chooses another number and the same procedure continues until there are no numbers at the end of the game. Students will have to think carefully about the numbers they choose so they can get a higher point total.	(E) This exercise can be completed using a utility program or with pencil computations. Computer Program: 2 PRINT "PRIME NUMBERS <50" 5 PRINT "2" 10 FOR X = 2 TO 50 20 FOR Y = 2 TO SQR (X) 40 IF X/Y = INT (X/Y) THEN 50 NEXT Y 60 PRINT X " "; 70 NEXT X (1) 10 consecutive numbers with two or more digits – what is the maximum number of primes? (2) Set of 10 consecutive three-digit numbers with four? With only three? With only two? None? (Answer: 100-109; 130-139; 160-169; 110-119; 120-120) (3) 11 and 101 are prime. Is 1 001? Check out 10 001, 100 001 and 1 000 001 on the computer. (Adapted from Arithmetic Teacher, Vol. 34, #5, January 1987.)

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 7. Lists the set of factors for whole numbers up to 200.	Develop a method for finding factors at a concrete and semi-concrete level before formal presentation. e.g., Use tiles to develop factors of a given number. Using blank cardboard tiles, students should make all possible rectangles with an area of 12 square units. Draw and label each rectangle as it is formed.  The dimensions of the various rectangles reveal the factors of the given number (12).	(E) Repeated division can be used to find factors of a number, as well as reducing a fraction to lowest terms and giving the L.C.M. e.g., 2(24 36 2(12 18 3(6 9 2 3 L.C.M. = 2 × 2 × 3 × 2 × 3 = 72 Reduce 24/36 2(24 / 36 2(12 / 18 3(6 / 9 2 / 3 $\frac{24}{36} = \frac{2}{3}$	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 7. Lists the set of factors for whole numbers up to 200. (cont'd)		Students may develop or use a simple program which generates a set of factors. (Arithmetic Teacher, Vol. 34, #5, January 1987, p. 36.)	See elective suggestions

Basic Program

```
100  PRINT "WHAT IS THE
     NUMBER";
110  INPUT N
120  PRINT "THE FACTORS ARE"
130  FOR K = 1 TO N
140      LET Q1 = N/K
150      LET Q2 = INT (N/K)
160      IF Q1 = Q2 THEN PRINT K
170  NEXT K
180  END
```

LOGO Program

```
T0  FACTOR :N
    MAKE "K 0
    PRINT (THE FACTORS ARE)
    CHECK :N :K
END
TO CHECK :N :K
REPEAT:N[MAKE "K:K + 1
    TEST REMAINDER :N:K = 0
    IF TRUE THEN PRINT :K
END
```

<table>
<tr><th>Objective</th><th>Clarification or Example</th><th>Elective Suggestions</th><th>Integration of Problem Solving and Technology</th></tr>
<tr>
<td>A. Whole Numbers
8. Expresses a number as a product of its prime factors.</td>
<td>Encourage the use of calculators and/or factor trees.
Repeated division on a calculator or using the repeated division method can be helpful. Students must remember that only primes can be used.
With a calculator, guess and check can be used to see what primes go into a given number.</td>
<td>(E) Have students look at numbers whose prime factorization consists of one prime factor;
e.g., 16 = 2 × 2 × 2 × 2 = 2^4.
All of the factors of 16 can be expressed as 1, 2^1, 2^2, 2^3 and 2^4. What is the relationship? (Five factors → one more than the exponent.) Have students experiment with other numbers.</td>
<td>Problem Solving: At a junior high school there are 1000 students and 1000 lockers. The lockers are numbered in order from 1 to 1000. A student entered the building and opened every locker. A second student closed every locker having an even number. A third student changed every third locker, closing those that were open and opening those that were closed. A fourth student changed the fourth locker, and so on. This continued until all 1000 students passed through the locker room. What was the position of locker #1000?
Strategies: Reduce to a simpler problem; e.g., What if there were only 20 students and 20 lockers?
Make a chart to organize the data.
<table>
<tr><th></th><th></th><th colspan="4">Students</th></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td></td><td>1</td><td>0</td><td></td><td></td><td></td></tr>
<tr><td>Locker #</td><td>2</td><td>0</td><td>0</td><td></td><td></td></tr>
<tr><td></td><td>3</td><td>0</td><td>0</td><td>C</td><td></td></tr>
<tr><td></td><td>4</td><td>0</td><td>C</td><td></td><td>0</td></tr>
</table>
Look for a pattern.
Extension: How many students changed the position of locker #600?</td>
</tr>
</table>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 8. Expresses a number as a product of its prime factors. (cont'd)		In each chart, what are the next five numbers after 99? In our number system, base ten, 9 is one less than 10. How does this fact make the pattern diagonal on the chart? (Arithmetic Teacher, Vol. 32, #7, March 1985, p. 39.)	

65

66

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 9. Uses a calculator or microcomputer to generate a set of multiples of a given number	Reinforce concept of multiples with a simple verbal activity. e.g., count by 2's count by 3's count by 5's Have them investigate <u>ways</u> they can generate sets of multiples on a calculator. e.g., addition, multiplication. Develop a program on the computer to generate multiples of a given number.	(E) Use the computer program to generate <u>common</u> multiples for two or more numbers. (R) Have students <u>use</u> utility programs to generate multiples. This is a simple BASIC program that will do this:	Include in development of the lesson. <u>Problem</u> 1) There are two sizes of tables in a banquet hall. One size seats exactly 5 people and the other size seats exactly 8 people. At tonight's banquet, exactly 79 people will be seated at less than one dozen tables, and there will be no empty places. How many tables of each size will there be? 2) The members of a flag squad wanted to arrange themselves into rows with exactly the same number of squad members in each row. They tried rows of 2, 3, and 4, but there was always one squad member missing. Finally they were able to arrange themselves into rows with exactly 5 in each row. What is the least number of members in the flag squad? (pp. 90-91, <u>Creative Problem Solving</u>, G. Lenchner, Houghton Mifflin Company.)

Clarification or Example:

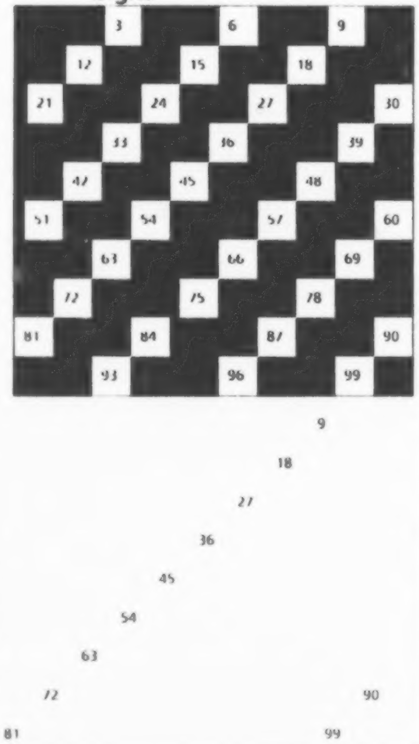
```
  5  REM MULTIPLES OF A NUMBER
 10  HOME
 20  INPUT "WHAT IS THE NUMBER THAT
     YOU WISH SEE THE MULTIPLES FOR?";
     N
 30  INPUT "HOW MANY MULTIPLES DO
     YOU WANT LISTED?"; M
 40  HOME
 50  PRINT "MULTIPLES OF "; M
 60  PRINT " - - - - - - - - - - - - - "
 70  FOR X = 1 TO M
 80  PRINT N * X,
 90  NEXT
100  END
```

Elective Suggestions:

```
100  REM  FIND THE LEAST
     COMMON MULTIPLE
110  READ A, B, C
120  LET X = A
130  IF INT (X/A) = X/A THEN 160
140  LET X = X + 1
150  GO TO 130
160  IF INT (X/B) = X/B THEN 190
170  LET X = X + 1
180  GO TO 130
190  IF INT (X/C) = X/C THEN 220
200  LET X = X + 1
210  GO TO 130
220  PRINT "THELCM OF"; A; B; C;
     "IS"; X
230  GO TO 110
240  DATA
250  DATA
260  END
RUN
```

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 10. Determines whether a number is divisible by 2, 3, 5, 6, 9 or 10.	Use the calculator to reinforce divisibility rules. Teachers may also wish to review divisibility rules with the use of charts.	(R) Form divisibility charts. Students can make posters to show which tables have ending rules or sum of digit rules or one that displays 2s, 5s, and 10s tables and another that displays 3s and 9s tables. Display the posters with questions underneath. Fig. 4. Divisibility rules: Ending rules 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78 80 82 84 86 88 90 92 94 96 98 100 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100	Develop a program which tests for divisibility. Problem 1. The pages of a certain book are numbered consecutively from 1 to 500. How many page numbers meet each of the following conditions? a) The page numbers contain the digit 5 and are divisible by 5. b) The page numbers contain the digit 5 but are not divisible by 5. c) The page numbers do not contain the digit 5 but are divisible by 5. 2. Replace the missing digit so that the resulting number is divisible by 9. a) 1 4 5 6 ☐ 28 b) ☐ 6 49 c) ☐ 54 321 Replace the missing digit so that the resulting number is divisible by 3.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 10. Determines whether a number is divisible by 2, 3, 5, 6, 9 or 10. (cont'd)		10 20 30 40 50 60 70 80 90 100 What tables are these? Look at the digits in the units place. Do you see a pattern? How can you recognize the numbers that belong in these tables? In each of these tables, what are the next five numbers after 100? Our number system is base ten; in base ten the 2s, 5s, and 10s tables have ending rules. How are 2, 5, and 10 related to the number 10?	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 10. Determines whether a number is divisible by 2, 3, 5, 6, 9 or 10. (cont'd)		Fig. 5. Divisibility rules: Sum of digits  What tables are these? Add the digits in each number. What pattern do you see in the answers?	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
B Decimals 1 Maintains previously developed skills with decimal numbers (place value, expanded and standard forms, adding, subtracting, multiplying and dividing decimal numbers).	Assess student proficiency in basic operations (see Objective #1, whole numbers for drill and practice activities). Encourage the use of varying computational strategies when reviewing basic operations. (Game: Adapted from Arithmetic Teacher, Vol. 32, #6, February 1985, p.56.) Calculating by Teams Objective: To practise computation with fractions, decimals, and percentages emphasizing speed and accuracy. Directions: 1. On an 8½" × 11" sheet of paper, create six questions in large writing like these (for decimals): a. 2.45 + 6.8 = □ b. □ − 0.027 = ○ c. 5.1 + 7 + ○ = Δ d. Δ − 10.023 = ▭ e. ▭ + 6.01 = ⏢ f. 2.69 + ⏢ = ______ Reproduce the sheets so that you have as many as the number of rows of six students. 2. Cut the activity sheets into strips containing one problem.	(R) Use base 10 blocks to reinforce place value basic concepts. (R) "Mad Minute" or "Math Warm-ups" can be used to increase student proficiency. (E) Discuss division of decimal numerals using a calculator. Ask students how they express the remainder when it is given in decimal form. How many are left over? Students could explore repeating remainders. (R) Fig. 4 $3\overline{)6.93}$	Houghton Mifflin MAC 7, Disk A

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
B. Decimals 1. Maintains previously developed skills with decimal numbers (place value, expanded and standard forms, adding, subtracting, multiplying and dividing decimal numbers). (cont'd)	3. Distribute the strips in order (a-f) down the row. When each student has a problem strip, give the signal to begin. The student with problem strip b cannot do the problem until the number that goes into the □ is passed on from the student with problem strip a. 4. The student with problem strip f brings the "final answer" to the teacher for checking. Students are encouraged to double-check their work and can signal at any time that a different answer is being passed back because of an error. The first row to submit the correct final answer is awarded one point for each player. *Going further* 1. Have students make up problem strips for future relays. The teacher will eventually have a nice supply for classes in future years. 2. Make up strips with only decimals, fractions, and so on, or combinations of operations. This activity can be adapted for younger students by creating strips with easier problems.		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
B. Decimals 1. Maintains previously developed skills with decimal numbers (place value, expanded and standard forms, adding, subtracting, multiplying and dividing decimal numbers). (cont'd)	3. Alternate methods for passing out strips to keep the competition fair (e.g., vertical or horizontal rows, front to back or back to front). Post a cumulative record of points earned. (From the file of Leona Burke Worth, Township of Ocean School District, Oakhurst, NJ 07755.) Write the digits in the boxes to make the largest and the smallest answers. Use each digit only once. Largest Answer — Smallest Answer 2 9, 3, 1, 6 □□ 7 + □ □ — □□ 7 + □ □ 3 3, 4, 7, 3 6□ □ - □ □ — 6□ □ - □ □ 4 8, 4, 7, 0 3 4□□ - 2 □□ — 3 4□□ - 2 □□ 5 2, 8, 6, 1 4 □□ × □□ — 4 □□ × □□ (Arithmetic Teacher, Vol. 34, #7, March 1987, p. 31.)		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
B Decimals 2 Compares and orders decimal numbers.	Fold a strip of paper repeatedly to form a decimal number line, giving students a tool to order and compare simple decimals. e.g., 0.5 0.25 0.5 0.75 0.125 0.250 0.375 0.500 0.625 0.750 0.875 Game Materials: Decimals written on 8½" × 11" sheets of paper (one number on each sheet). Directions 1. Give a decimal to each class member. 2. Divide the class into teams of three to five students each. 3. Each team must go to a separate part of the room and arrange its members in a line, with the member holding the smallest number in the front and other team members ranked behind in the order of the numbers held. 4. The first team to arrange itself in the correct order wins.	(E) Extend the folding experiment and have students address the concept of infinity between two points. Teachers may wish to develop dot to dot games which order decimals from least to greatest. (Arithmetic Teacher, Vol. 34, #7, March 1987.)	Develop or use a program to order/compare decimals.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
B Decimals 2 Compares and orders decimal numbers. (cont'd)	5 The team size can be increased to make the game more difficult. This ability can be modified for lower grades to teach several concepts involving ordering.		
3 Rounds decimal numbers	Give real life examples to demonstrate the need for rounding decimals. e.g., 1 litre of gas @ 33.7¢ 25 litres of gas ? Could you pay for this and get exact change back? Encourage estimation and the use of calculator to check answers. Have students think of other examples in everyday situations where decimals are rounded. (See Journeys in Math 7 TRM, p. 38.)	(R) Give students a problem in which they would have to round the answer to increase their understanding. e.g., Plane flying from Gander to London. Halfway across the ocean the plane develops problems. Should the plane go back or continue the journey? (E) See Journeys in Math 7 TRM, p. 39.	

<table>
<tr><th>Objective</th><th>Clarification or Example</th><th>Elective Suggestions</th><th>Integration of Problem Solving and Technology</th></tr>
<tr>
<td>C. Fractions

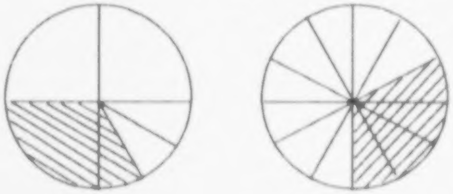
1. Maintains previously developed skills with fractions (concept of a fraction, need for fractional numbers, equivalent fractions, basic fractions) at a concrete level.</td>
<td>Teachers may have students use strips of adding machine tape as a measuring unit and have them measure various objects and find the need to subdivide their unit, and discuss the concept of a fraction.

Students could fold their measuring unit into 2, 4, 8, 3, 6, etc., and label as directed.

Fraction Tape
<table>
<tr><td>0</td><td></td><td></td><td></td><td>1</td></tr>
<tr><td>$\frac{0}{2}$</td><td></td><td>$\frac{1}{2}$</td><td></td><td>$\frac{2}{2}$</td></tr>
<tr><td>$\frac{0}{4}$</td><td>$\frac{1}{4}$</td><td>$\frac{2}{4}$</td><td>$\frac{3}{4}$</td><td>$\frac{4}{4}$</td></tr>
</table>
Using this unit they should identify equivalent fractions.

Students may also use multiple boards to identify equivalent fractions.

e.g., 1/3 = ?
<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td><td>18</td><td>21</td><td>24</td><td>27</td><td>30</td></tr>
</table>
e.g., 1/3 = 3/9, etc.</td>
<td>(E) Develop fraction dominos to reinforce equivalent fractions.
<table>
<tr><td></td><td>4/8</td><td></td><td>1/2</td><td>1/3</td><td></td><td>2/6</td><td></td></tr>
</table>
(R) Continue to allow students to use fraction tapes/multiple boards until the concept is fully understood.</td>
<td>MAC 7, Disk B.</td>
</tr>
</table>

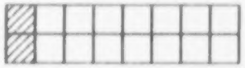
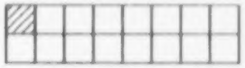
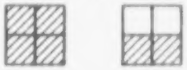
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
C Fractions 2. Identifies mixed numbers and improper fractions and converts from one to the other.	Students should develop this concept in pictorial form. Pizza  Divide amongst three people equally. Each person receives 1 1/3 pieces. Divide amongst three people equally. Each person receives 4/3 pieces. After some examples, students should develop the rules for conversion. If students experience difficulty with this at the pictorial level it can be explained to them using concrete manipulatives; e.g., fraction tape. (See Journeys in Math 7 TRM, p. 172.) This concept should be developed until students can carry out the operation at an abstract level.	(R) Encourage the use of manipulatives. Paper circles, pizzas, chocolate bars can be quite effective to illustrate this concept.	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology

C. Fractions
3. Orders fractional numbers.
	Use the measuring unit from Objective #1 to order fractional numbers (limit to those fractions shown). Use multiple boards to compare fractions.
	e.g., compare 1/2 and 4/6
	Form 1/2 1 2 3 4 5 6 7 8 9 10
	  2 4 6 8 10 12 14 16 18 20
	Form 4/6 4 8 12 16 20 24 28 32 36 40
	  6 12 18 24 30 36 42 48 54 60
	Since 3/6 < 4/6
	then 1/2 < 4/6
		(E) Teachers may wish to use fractions with unusual denominators wherein the order is not as obvious.	Some students may write a utility program and others may use one to order fractions. This will also generate a group discussion on conversion from fractions to decimals.
			MAC Courseware 7, Disk B.

4. Uses concrete manipulatives to demonstrate the addition and subtraction of fractions with and without common denominators.	While many students are able to use a "rules" approach to operating with fractions, there is much evidence to indicate that fewer students understand the operation. A number of manipulatives may be used to demonstrate addition and subtraction. Cuisenaire rods - Teachers and students should be familiar with the relationships among the various lengths and colours of the rods. (Instructions and activities are usually included when Cuisenaire rods are purchased.)	(R) A multiplication table produces a multiple board for fractions, from which equivalent fractions can be determined. e.g., 1 2 3 4 5 6 ... 2 4 6 8 10 12 ... 3 6 9 12 15 18 ... 4 8 12 16 ... $\frac{1}{2} + \frac{2}{3}$ $\frac{3}{6} + \frac{4}{6} = \frac{7}{6}$	"Fraction Challenge" MAC Courseware 7, Disk B 90

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
C Fractions 4 Uses concrete manipulatives to demonstrate the addition and subtraction of fractions with and without common denominators (cont'd)	Fraction bars or fraction circles: these inexpensive cards can easily be constructed or found in commercial materials as black-line masters. Students should colour and cut out their circles, using a common colour for each fraction (e.g., thirds – green, quarters – blue). Adding fractions with common denominators (same colour) is straightforward, but students will be challenged to describe their answer when adding $\frac{1}{4} + \frac{1}{3}$ (1 blue + 1 green). A common denominator (common colour) must be found to describe the sum.		
5. Writes number sentences to describe the addition and subtraction of fractions.	The intention here is for students to demonstrate their understanding of the operation. It is not necessary for students to solve the question but to be able to write down what is being demonstrated with the concrete manipulatives. This objective directly relates to the previous one and should be tied to the teaching of that objective.		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
C Fractions 6 Uses concrete manipulatives to demonstrate the multiplication and division of proper fractions.	**Multiplication** Illustrate this concept using a chocolate bar. e.g.,  Eight friends share this chocolate bar equally. Shade the portion each person receives. Bill decides to eat 1/2 of his share. Shade the portion he eats.  Students demonstrate their understanding by writing a number sentence. 1/2 of 1/8 = 1/16 (Discuss the meaning of the word "of".) **Division** Review the concept of division as repeated subtraction, then ask "how many times can 1/4 be subtracted from 1 1/2?"  Students demonstrate their understanding by writing a number sentence. (See Journeys in Math 7 TRM, pp. 186-189.)	**(R)** Continue allowing students to use concrete manipulatives or pictorial form to develop full understanding.	"Fraction Challenge II" MAC Courseware 7, Disk B. Problem A farmer died leaving 17 cows. According to the terms of his will the eldest child was to receive 1/2 of the cows, the second child to receive 1/3 and the youngest child to receive 1/9. The children were puzzled about how to carry out the terms of their father's will since none of these fractional parts of 17 cows was a whole number. Finally, a generous neighbour offered to loan a cow to the children. They then had 18 cows: 1/2 of 18 was 9 cows; 1/3 of 18 was 6 cows; and 1/9 of 18 was 2 cows. The total 9 + 6 + 2 was the original 17 cows. The 18th cow remained to be returned. How was this possible? (See Journeys in Math 7 TRM, pp. 186-189.)

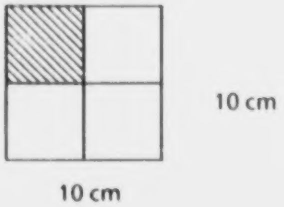
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
C Fractions 7. Writes number sentences to describe the multiplication and division of fractions.	This objective should be taught with the previous objective. Students need only describe the concrete operations using number sentences, and should not be encouraged to compute at a formal level.		
D Integers 1 Maintains previously developed skills with integers (concept of integers, need for integers, ordering of integers).	Demonstrate the need for integers using: • change in temperature • balancing chequebooks • above and below sea level • golf scores of above and below par • time zones.	Remedial: Have students keep a temperature log for a week. Discuss daily changes; for example, Monday to Wednesday, or other variations.	MAC 7, Disk C Calculator Activity – Holtmath 7 Teacher's Edition, p. 257
2 Uses concrete manipulatives to demonstrate the addition of integers.	(Concrete Activities, Journeys in Math 7 TRM, pp. 292-303.)	(E) Journeys in Math 7 TRM, p. 299	Holtmath 8, Disk 5
3. Writes number sentences to describe addition of integers.	This objective should be combined with the previous objectives. Students should write number sentences to describe the concrete activity (adding integers). Adding integers at a formal level (development of addition rules) is discouraged.		

RATIO AND PROPORTION

Grade 7

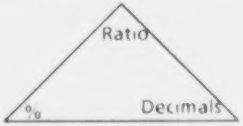
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1 Applies and practises problem-solving skills in new situations.	The intent of this objective is that every new situation (especially if it is perplexing), be approached from a problem-solving perspective. Every opportunity to teach a new problem-solving strategy should be taken when developing or reviewing concepts in this strand.		
2. Maintains previously developed skills (identifies ratios as ordered pairs of numbers related to concrete situations; uses whole number constants to generate equivalent ratios).	Concrete Activities, Journeys in Math 7 TRM, pp. 204-207. Holtmath 7 Teacher's Edition, p. 191 "Alternative Teaching Strategies".	(R) Encourage the extended use of manipulatives for remediation. (E) Have students develop their own parallel concrete activity.	
3. Uses concrete manipulatives to construct ratios in the following forms: *a*:*b*, *a* as to *b*, and $\frac{a}{b}$	The ratio $\frac{3}{6}$ can be expressed as 3:6 and 3 as to 6. Ensure that these forms are used interchangeably. When evaluating the objective, present ratios in all three forms. Holtmath 7 Teacher's Edition, p. 191 Journeys in Math 7 TRM, p. 204.		

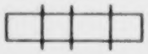
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Verifies the equivalence of two ratios using common multiples or factors: e.g., $\frac{14}{6}\begin{matrix}(\div 2) \rightarrow \\ (\div 2) \rightarrow\end{matrix}\frac{7}{3}$	Concrete Activities, Journeys in Math 7 TRM, p. 206 Holtmath 7 Teacher's Edition, p. 193 "Alternative Teaching Strategies"	(R) Students maintain facility at a concrete level. See Holtmath 7 Teacher's Edition, p. 193.	MAC 8 (9A) Ratio Rendezvous
5. Finds a missing element of a proportion using a common multiple of the elements: e.g., $\frac{3\,(\times 3)}{4\,(\times 3)}\begin{matrix}\rightarrow \\ \rightarrow\end{matrix}\frac{x}{12}$	The intent is to encourage the development of equivalence rules. (Do not allow the use of cross products.) Concrete Activities, Journeys in Math 7 TRM, p. 210.	(R) Students maintain facility at a concrete level (see #2). (E & R) Journeys in Math 7 TRM, p. 211.	

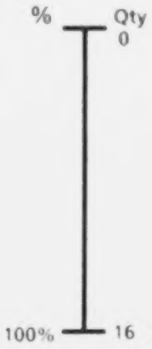
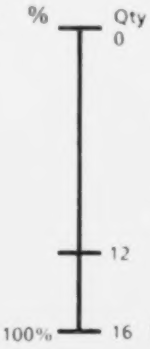
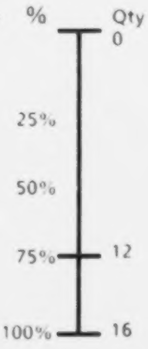
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
6 Identifies percent as a ratio: e.g., (p:100 or $\frac{p}{100}$)	Use 10 x 10 cm grid paper. Have students calculate the total number of squares. Shade a portion and write the ratio of the shaded portion to the whole. Write the percentage of the shaded portion to the whole. Have students develop multiple examples to generate a rule for writing a percentage as a ratio.  e.g., Total 100 Shaded 25 Ratio $\frac{25}{100}$ Percent 25% $\therefore \frac{25}{100}$ is 25%	(E) Have students estimate percents on test scores. Have them check on their calculators. (E) Explore the origin of the root word "cent" e.g., century (R) Journeys in Math 7 TRM, pp. 236-237.	

RATIO AND PROPORTION

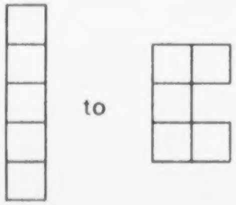
Grade 7

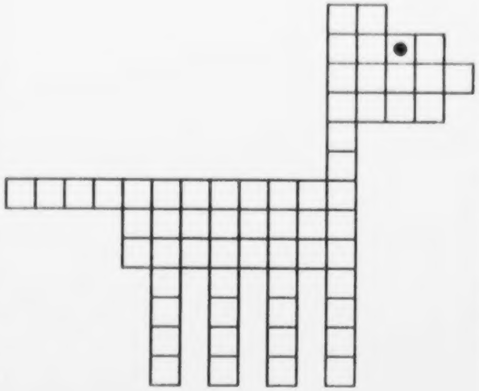
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
7. Expresses ratios as percents and decimals and vice versa (limit: ratios in the form a:b, where b = 2, 4, 5, 10, 20, 25, 50): e.g., $\frac{3}{4} \rightleftharpoons \frac{75}{100} \rightleftharpoons 75\%$	Emphasis should be placed on the notion that numbers have different equivalent forms. Concrete Activities, Journeys in Math 7 TRM, p. 240. The emphasis should be on simple ratios and mental conversions. Drill cards will develop ability to convert from one to the other.  Cover one vertex and have student verbalize what is missing. Extend this by covering two vertices.	(R) Journeys in Math 7 TRM, p. 241. (Journeys in Math 7 TRM Teaching Aids Game 12, p. 76.)	Students should use their calculators to find percents of their own test scores.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
8 Finds the percent of a number: e.g., 15% of 25	To give meaning to this objective, some background preparation is necessary. Students should demonstrate their understanding of percent in concrete terms. e.g., Shade: 50% of a pie 100% of a pie 75% of a pie 25% of a pie 10% of a pie 1% of a pie e.g., Money 50% of $1.00 = 50¢ 25% of $1.00 = ______  Students form the rule. How is the % of a number calculated?	(E) Journeys in Math 7 TRM, p. 243.	Estimate percentages of numbers. Check using calculators. Develop utility program to find percentages of numbers.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
9. Expresses one number as a percent of another number: e.g., 12 is what percent of 16? or $\frac{12}{16}$ = ______%	Discuss in terms of real life examples. Where would the percent of a number be used? e.g., test scores sports results Emphasis can be placed on estimation and the use of the calculator/computer to check the result. e.g., 12 is what % of 16?  Fig. I - Place 16 opposite 100%  Fig. II - Estimate and position 12  Fig. III - Subdivide scale as necessary (50%, 25%)	(E) The sale price of an article is $12. If the original cost was $16, what percentage was saved during the sale?	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.			
2. Maintains previously developed skills.	Introduce concept of measurement. Use examples such as labels on food packages, speedometer numbers, sporting event distances. Measurement should be taught as "a comparison to some arbitrary unit". These units are repeated and may be combined into larger units. The need to subdivide a unit should be demonstrated when a fraction of a unit is needed to describe a length, mass, etc. Initially the arbitrary unit may be one's hand. The need to standardize units (metres) should evolve.		
a) linear	Estimate and measure objects in the classroom such as the length of a paper clip, height of a door, or length of a piece of loose-leaf paper. Have students estimate and measure the length of given line segments. Have students draw line segments, estimating and measuring. When measuring they can express the units (in m and cm).		Problem Using maps and distance charts to solve problems. Students will need to interpret maps and charts, and plan using more than one possible path to a solution.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Maintains previously developed skills. (cont'd) b) perimeter	Estimate and measure the distance around various rectangular and triangular objects.	(E) Explore how rearranging 5 square cards changes the perimeter. e.g., 	Problem • Explore perimeters of compound figures. e.g.,
c) area	Using a ruled graph paper, cut out rectangles and triangles then determine their area by counting squares. Have students develop a strategy (such as a formula) for determining the area of rectangles or triangles.	(E) Computer: Mathematics 7 Activities Courseware, Metric Mysteries, Houghton Mifflin. **Remedial** *Create a Creature: Designate an area, such as 60 cm^2. Have the children cut out the area of squared paper into as many sections as they wish and then arrange the sections to form a creature. They will also enjoy creating creatures with the areas of certain parts specified. For example, they might use 24 cm^2 for the body, 16 cm^2 for the legs, and 20 cm^2 for other parts. Such a fanciful creature is shown on the next page.	• Using a variety of tools such as string, grid paper, ruler, etc., have students try to determine the area of classroom objects such as a book front, desk top. • A square decimetre is a handy size for comparing with other objects. Have each child cut a square decimetre (10 cm × 10 cm) from squared paper. Specify several objects and ask them to compare the area of each to the square decimetre. They might use objects such as their textbook covers, a single surface of their crayon boxes, the area of their hands (outlined on paper), or the area of their desk tops. Lead them in a discussion of strategies for estimating area.

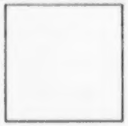
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Maintains previously developed skills. (cont'd) c) area		A final project might be making creatures of any size and then calculating the area of the finished cutouts. 	They might lay their square decimetres over an object whose area they want to estimate to see if the object's area is greater than, less than, or about equal to that of the square decimetre. They might see approximately how many times the decimetre square can be placed on a large area. They might also lay a smaller object on top of the square decimetre and estimate the fraction of the square decimetre that is taken up by the object. *(Arithmetic Teacher, Vol. 31, #4, December 1983, pp. 4-11.)

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Maintains previously developed skills. (cont'd) c) area		*Relating perimeter and area Instruct children to cut out as many rectangles and other polygons with a given area as they can. They should then figure the perimeter of each polygon and put the polygons in order from smallest to largest in perimeters. The class should share their results and list as many solutions to the problem as possible. Next, have students cut out four rectangles, each having an area of 48 cm^2. They will realize that the shapes of these can vary. After they have cut out their four rectangles, invite them to cut each of them into two or more pieces to form another polygon. They can rearrange the pieces to form L-shaped figures (hexagons) and other polygons. Ask them to find and label the perimeter of each polygon. One-centimetre segments can be counted to find the perimeter for some polygons; for others in which the squared paper has not been cut at right angles, rulers can be used to find the lengths of the sides, or the perimeter of the polygons can be estimated. *(Arithmetic Teacher, Vol. 31, #4, December 1983, p. 11.)	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Maintains previously developed skills (cont'd) d) volume	Have students construct rectangular prisms with centimetre cubes, and develop a strategy for determining the volume. Extend this to practical situations such as the volume of an aquarium.		Problem Have students explore what happens to the volume of a cube if the length of a side is doubled. Use a calculator, prepare a chart.
e) capacity	In general, capacity units are used for liquids. The relationship of 1 mL = 1 cm^3 can be explored by using a waterproof rectangular solid, comparing the water it can hold to the cubic centimetre blocks it can hold. Explore the use of capacity in practical situations such as the capacity of a fuel tank, a pop bottle, a swimming pool, a grain bin.		
f) mass	The mass of an object is "the amount" of material in that object. Two common mass units are the kilogram and gram. The feel of these can be found by estimating masses of objects by picking them up and then finding their actual mass on a balance. The appropriateness of other SI units (milligram, tonne) should be explored.		

117

118

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Maintains previously developed skills. (cont'd) g) angle	Estimate angles, and then measure using a protractor. Some concrete examples are the hands of a clock, the corner on a table, the blades of scissors. Discuss the inside and outside scale on the protractor.		Problem How do you measure an angle whose rays are too small, or the angle that is greater than 180°?
h) transformational geometry	Identifies and draws translations (slides), reflections (flips) and rotations (turns) of figures, and tests for congruence.	(E) Explore tiling patterns (tessellations). Using shapes, try to tile a plane or design a tessellation with one or more shapes. (Arithmetic Teacher, Vol. 31, #5, January 1984, p. 54.)	
3. Expresses equivalent measures of SI units (linear).	The appropriateness of units should be explored. For example, the distance from the school to home should be measured in metres or kilometres but not centimetres. Conversions used in practical situations should be developed.	(E) Explore through concrete situations the relationship of SI units of area, volume, and mass. (R) Arrange students in groups. One group will draw lines of, for example, 5.2 cm, 6.5 cm, 7.8 cm, 15.0 cm. Another group will make line segments of 52 mm, 65 mm, 78 mm, 150 mm. Groups then try to find a match.	Computer-Mathematics Activities Courseware 7, Houghton Mifflin, Congruent Figures, p. 63. Problem-Solving Activity:  Draw 2 lines so the picture shows 4 congruent triangles. (Arithmetic Teacher, Vol. 31, #5, January 1984, p. 31.)

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Understands and uses the terms similar and congruent with respect to geometric figures.	Congruence is having the same size and shape. Similar is having the same shape but not necessarily the same size (apply transformational geometry). Students can be encouraged to draw congruent and similar shapes. Journeys in Math 7 TRM, pp. 355, 378-379. Holtmath 7 Teacher's Edition, pp. 332-333.		
5. Understands and uses the terms symmetry with respect to geometric shapes.	Explore lines of symmetry by dividing a given shape into two congruent parts by folding or by using a mira. Using a mira, draw the reflection in the mira and make a new shape with a line of symmetry. This can be reinforced by transformational geometry (flips). The lid of a square box can be turned four different ways to fit the box. It has turn symmetry. The order of turn symmetry for a figure is the number of times it fits onto itself in one full turn. The order of turn symmetry for the square box is 4. Have students explore turn symmetry by tracing shapes and turning them to find their order of turn symmetry.	(E) Discuss graphic design by M.C. Escher. Produce patterns with turn symmetry.	

121

122

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Understands and uses the term symmetry with respect to geometric shapes. (cont'd)	Lines of symmetry and turn symmetry can be reinforced by examining trademark designs of companies such as Shell Oil, Westinghouse Electric, Ralston Purina, Chrysler Corporation, or Mattel. A strategy to develop students' interest is to create an attractive bulletin board displaying examples of trademarks with line symmetry, turn symmetry, as well as combinations. Some non-examples might also be included.		
6. Constructs geometric designs using tools such as a computer, compass, straightedge, ruler or mira.	Some possible designs are company logos, tessellations of the plane (M.C. Escher). By exploring various designs, students learn the use of geometric tools and explore properties of geometric figures. Encourage students to be creative. One way is to arrange colourful and attractive bulletin board displays of geometric designs. (See Journeys in Math 7 TRM, p. 149.)		A LOGO computer program may be used to create designs. Journeys in Math 7 TRM, pp. 117, 120.

123

124

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations. 125	The problem-solving framework should be re-emphasized: understanding the problem, developing a plan, carrying out the plan, and looking back. Problems should not be limited to traditional word problems, but should include situations from the student's environment as much as possible.		Objectives can be designed to incorporate problem solving into data management, such as: a) The student would ask questions in which statistical data would help to find an answer. b) The student would decide what type of statistical measure would help to find the answer. c) The student would collect and compute the necessary data. d) The student would organize and interpret the data. e) The student would answer the question. f) The student would recognize the other choices of data and other interpretations, and should therefore be ready to defend or modify the conclusions. 126

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2 Demonstrates a knowledge and understanding of the use and purposes of statistics as it affects daily living.	The students can use newspapers to locate information such as weather reports, prices of used cars, sports statistics, and fashion prices. They should then display this information and find the high price, low price and "typical" price. The students should be able to answer the following questions: a) How do numbers affect our daily lives? b) What do the numbers compare? c) List the possible sources of data: e.g., newspapers, magazines, books, radio, television, personal experiences, sports cards, opinion polls, local surveys, etc. d) Why are statistics kept? e) How do statistics help us? f) How are statistics displayed clearly? By relating to the students' environment, discuss how numbers are used in their daily lives. Use the above sources to explain what is being compared, why statistics are kept, how they help us, and how the results can be displayed (tables, graphs, etc).	(E) Investigate what a statistician is, and report what a career as a statistician would be like.	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Collects and records data (tally sheets and frequency tables). 129	Use questions which require the students' output such as: a) How many kilometres (or blocks) is your home from school? b) What is your hair colour? c) What is your eye colour? d) What type of shoes are you wearing now? e) What is your favourite sport? f) Research data in the school library. Use the tally method of fives (卌) to record the number of occurrences, and then write down the frequency counts. The students answer questions concerning the greatest, smallest, and most typical occurrences. Use a month's temperature data for a town or city to find a way to organize the high and/or low temperatures into frequency tables, means, and ranges.	(E) Using data from newspapers have the student make frequency tables, make up five questions on the data, exchange the data and questions with another student, and answer each other's questions. (E) Use 49 cards (labelled 1 to 49) and play "Lotto 6-49" 20 times (draw 6 numbers each time), and record the results. (Journeys in Math 7 TRM, p. 321.)	130

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Understands and uses the term average (mean) as related to practical situations (e.g., test marks).	Use information such as bowling scores, test scores, students' weights, etc. The students will determine averages (means) by adding all the items (maximum of three digits per number) and dividing by the number of items. Have groups of students list and compare items (e.g., number of hours of TV watching, ages in months, etc.). (Journeys in Math 7 TRM, p. 336.)	(R) Use smaller numbers and restrict sample size to determine mean. (E) Find the missing mark in a list of marks when the mean is given.	Write a computer program that will calculate the mean for any list of numbers. Houghton Mifflin MAC 8, The Three M's. Computer Average Program, Holtmath 7 Teacher's Edition, p. 301.
Extended Content Find the mean from a frequency table.	Use a list of students' weights to make a frequency table, and then use the frequency table to calculate the average.		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Interprets data from pictographs, bar graphs, line graphs and circle graphs.	A graph displays and compares information. A pictograph uses symbols to display the data conveniently. A bar graph uses intervals and scales to display data. A line graph displays the relationship between two changing quantities. A circle graph represents data as being part of a whole. Using graphs from any available sources, the students should be able to identify the types of graphs and to read information from each graph.	Pictographs (R) Practise skip counting by 25, 10 or whatever unit used in the pictograph. Bar graphs and line graphs (R) Use a square corner of plastic to align the horizontal and vertical scales, and check to see that students are reading the scales correctly. Circle graphs (R) Review measuring angles in combination with fraction circles.	
6. Understands when and how to represent data in the form of pictographs, bar graphs, line graphs and circle graphs.	Each type of graph has its strengths and weaknesses. Pictographs are easy to read and appealing to look at but are not always very accurate. A bar graph is more accurate but is less appealing to look at. A line graph is best for displaying the relationship between two changing quantities, but has limits to its accuracy and care must be taken in making predictions based on trends.	Pictographs (E) Make pictographs that compare large numbers. (R) Use topics which lend themselves to pictorial representation.	Houghton Mifflin MAC 7, Pie Graphics. Use LOGO to construct graphs.

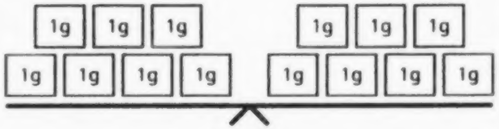
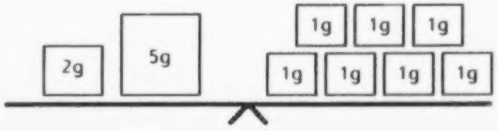
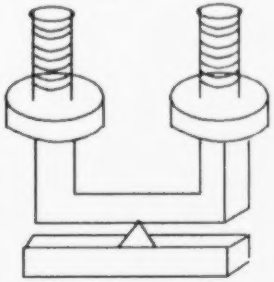
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
6. Understands when and how to represent data in the form of pictographs, bar graphs, line graphs and circle graphs. (cont'd)	A circle graph is good for displaying data as parts of the whole but can be more difficult to compare with other graphs. The students can use data sources (such as newspapers, surveys or from previous exercises) to draw various types of graphs. The students should use knowledge of the strengths and weaknesses of each type of graph to select the form in which the data should be represented. The students should be introduced to the steps in drawing a circle graph: a) find the angle measure by multiplying each percent times 360° b) construct a circle and use a protractor to draw each angle c) label the graph.	Bar graphs (E) Discuss when to use horizontal bar graphs and when to use vertical bar graphs. (E) Use scales that vary (don't start at 0). Line graphs (R) Use a line graph from a source to write a report describing what the graph is about and then reproduce the graph. (E) Draw a double line graph (e.g., high/low temperatures, populations of Calgary and Edmonton over a period of years). Circle graphs (R) Review how to find percents of numbers. (E) Given a circle graph with unlabelled sectors, have the students calculate what percent each sector represents	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.	Specific problem-solving techniques and skills will be developed with each objective.		
2. Understands and uses the term variable and uses variables to describe a concrete situation (e.g., number of jelly beans in a jar).	Use concrete situations to demonstrate the meaning and use of variables (a letter or symbol that represents an unknown number). For example, a variable can be used to express the number of pennies in a jar, or the age of a teacher.		Many BASIC and LOGO programs use variables in input statements. By varying the input value, the students can explore the effects. For example: LOGO Program TO SQUARE : SIDE FD : SIDE RT 90 FD : SIDE RT 90 FD : SIDE RT 90 FD : SIDE RT 90 END SQUARE 30 draws a square with side length of 30, SQUARE 40 draws a square with side length 40, and so on.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Uses variables to write mathematical expressions to represent practical situations (e.g., age of the students in the class in three years will be x + 3 years).	With the inclusion of the operations +, −, × and ÷, we can represent practical situations using open mathematical expressions. For example, when three pennies are added to a jar containing an unknown number of pennies, we can express this by x + 3. Use similar procedure for other simple expressions: three less than a number: x − 3 5 times my age: 5 × x or 5x. Extend the activity to more complex statements: three more than 5 times a number: 5x + 3. Give students an expression such as 3a − 2 and have them state its meaning in words.		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4 Evaluates expressions for given values of the variable (limit: whole numbers, positive rationals).	Expressions may be evaluated mentally, formally by substitution or organized in the form of tables. Use practical situations to reinforce the use of mathematical expressions. For example, movie admission is $2.50. A table is one method of showing costs for different numbers of people. Number — Cost 1 — $2.50 2 — $5.00 3 — $7.50 p — $2.50 × p The expression for cost is $2.50 × p or $2.50p.		Looking for a pattern is one problem-solving technique. Problem There are 18 animals in the barnyard. Some are chickens, some are cows. There are 50 legs in all. How many are chickens and how many are cows? By making a table, and examining data, a pattern emerges. Chickens 17 16 15 Cows 1 2 3 Legs 38 40 42 Use a calculator's automatic constant feature for ×, ÷, + and −. For each operation have students determine which operation is the automatic constant for their own calculator.

<table>
<tr><th>Objective</th><th>Clarification or Example</th><th>Elective Suggestions</th><th>Integration of Problem Solving and Technology</th></tr>
<tr>
<td>4. Evaluates expressions for given values of the variable (limit: whole numbers, positive rationals). (cont'd)
Extended Content
Find the mean from a frequency table.</td>
<td>From a table of values, students try to determine the defining expression by looking at the pattern.
<table>
<tr><td>x</td><td></td></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>5</td></tr>
</table>
The defining expression is x + 2.</td>
<td></td>
<td>Houghton Mifflin MAC 7.
Problem
For the normal range of summer temperatures the number of chirps made by a cricket in a minute is predictable. For example, at 16°C the cricket chirps 140 times per minute. At 24°C the cricket chirps 172 times per minute. What is the temperature at which the cricket chirps 180 times per minute? At 21°C, how many times will the cricket chirp per minute?</td>
</tr>
<tr>
<td>5. Uses variables to write mathematical sentences to represent practical situations (e.g., people in a classroom = boys + girls + teachers or p = b + g + t).</td>
<td>With the basic operations (+ , −, x, ÷) and the inclusion of = , we can represent practical situations using closed mathematical sentences.
Mathematical sentences (equations) may have one unknown (i.e., a number plus 3 equals 7 becomes x + 3 = 7) or have more than one unknown (i.e., students in the room = boys + girls becomes s = b + g).</td>
<td></td>
<td></td>
</tr>
</table>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
6. Uses concrete manipulatives to demonstrate the concept of "equals" (i.e., equality).	Using a balance scale and one gram masses, students should experiment to discover balance by manipulating the placement of the one gram masses. Having discovered that an equal number of one gram masses is required on each side of the balance, students should replace the one gram masses on one side of the balance with different size masses but maintaining balance.  7 = 7  2 + 5 = 7 Thus students should discuss the concept of equality as a state of balance. For example: 7 = 7 2 + 5 = 7 3 + 4 = 7 1 + 6 = 7		Problem You have a pile of 24 coins. Twenty-three of these coins have the same weight and one is heavier than the others. Your task is to determine which coin is heavier. You are given a balance beam which will compare weights. Develop a strategy to find the heavier coin in a minimum number of weighings. 

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
6. Uses concrete manipulatives to demonstrate the concept of "equals" (i.e., equality). (cont'd)	The forms used are limited to one unknown using whole numbers, positive fractions and positive decimals. Considerable time should be spent on the concept that an equation is a balance in which the left side equals the right side. A possible manipulative is a balance scale.		
7. Uses estimation, and guess and test procedures to solve equations of the form: $x + a = b$, $ax = b$, $ax + b = c$, and $\frac{x}{a} = \frac{b}{c}$	Solve $2x + 3 = 15$ by guess and test Try $x = 5$ $2 \times (5) + 3 = 13$ too small Try $x = 6$ $2 \times (6) + 3 = 15$ ✓ This reinforces substitution of expressions and organizing work to document the process.	(E) A simple classroom game is to have a student pick a secret number. Ask the student to perform some operation on that number (e.g., add 2) and then tell the class the result. Have class members find the secret number. This can be expanded to writing the equation and checking the solution. The game can be expended to include more than one operation and the sequence of the operations. (R) "Roll It". Prepare cards with equations having whole number solutions between 2 and 12. For example, $\boxed{x + 1 = 7}$ $\boxed{5b = 15}$	Use of calculators as a means of guess and test. The automatic constant function is a valuable aid to guess and test. e.g., To solve $\frac{x}{5} = 4.2$ $x = 10$ $10 \div 5 =$ 2 too low $x = 15$ $15 =$ 3 too low $x = 20$ $20 =$ 4 too low $x = 23$ $23 =$ 4.6 too large $x = 21$ $21 =$ 4.2 ✓

147

148

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
7. Uses estimation, and guess and test procedures to solve equations of the form: $x + a = b$, $ax = b$, $ax + b = c$, and $\frac{x}{a} = \frac{b}{c}$ (cont'd)		Deal 5 cards to each player. The first player rolls a pair of dice. Each player who has a card with an equation whose solution matches the dice discards that card. A discard may be challenged. If the discard was not correct, the player must pick up the discard and also take one from the challenger. The first player with no cards wins.	
8. Verifies solutions to equations by substitution.	Verification of a solution is simply trying to determine if the left side balances the right side. Is 6 the solution to $2x + 3 = 13$? $2x(\ ) + 3 = 13$? $2x(6) + 3 = 13$? $12 + 3 = 13$? $\therefore 15 \neq 13$ (Journeys in Math 7 TRM, pp. 276-277.)	(E) Journeys in Math 7 TRM, p. 277.	Encourage the use of a calculator. This reinforces the order of operations.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
9. Given ordered pairs, plot points on a coordinate plane.	This topic should be familiar to students and so need not be dealt with at great length. An interesting activity is to plot points in all 4 quadrants that determine a picture or design.	(E) Give students simple pictures or designs and have them transcribe it to a coordinate plane then to identify the points needed to reproduce their picture. Have students exchange with a classmate for checking by reproducing the picture from the given points.	Holtmath 7 Teacher's Edition, pp. 314-315. Software Disk 6, Graphing Ordered Pairs.

GRADE 8
MATHEMATICS

PLEASE NOTE

The following resources have received basic status and are available from the Learning Resources Distributing Centre:

Student texts for JOURNEYS IN MATH 7, 8
Student texts for HOLTMATH 7, 8, 9

Any other titles mentioned in this guide have not received formal status approval. They are presently under review and cannot, at this time, be obtained through the Learning Resources Distributing Centre.

CURRICULUM GUIDE (DRAFT)

PROBLEM SOLVING **Grade 8**

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Demonstrates an understanding of a problem-solving situation.	Problem solving should not be viewed as an isolated activity but rather as a process that is to be an integral part of the teaching philosophy to be used in the development of the other strands. The framework for problem solving should be introduced at the beginning of the year (suggested time – 3 to 5 periods). Brainstorm for a definition and examples. The following ideas should evolve about a problem: a) it has no readily apparent solution or the means to the solution is not immediately evident b) it can cause a person to be temporarily perplexed c) it may have no answer, one answer, or more than one answer	Teachers must recognize that problem-solving skills are essential for all students and that being perplexed when first encountering a problem is normal. Problems presented to students should be challenging yet solutions must be attainable to insure that students experience success. It is very important for teachers to realize individual student differences in learning; therefore the growth expectation should also vary. Individual needs can often be met by changing the conditions of a problem to make it simpler.	a) The use of calculators in problem solving must be encouraged so that time spent on tedious calculations is decreased and feedback on strategies is faster. Numbers from realistic and relevant situations are less imposing if calculators are used. b) Group work should often be used in problem solving. A student in a group deals with ideas and questions from other members of the group, and this may help each student to progress in developing problem-solving strategies.

Note: (E) = Enrichment
(R) = Remediation

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Demonstrates an understanding of a problem-solving situation. (cont'd)	d) it can be of a practical, everyday, personal or social nature as well as of a mathematical nature. (See Journeys in Math 7 TRM, p. 30.)	Manipulatives can also be used to meet individual needs. e.g., Students are given a pile of 21 markers. Two players are involved and take turns removing one, two or three markers. The winner is the player who removes the last marker. The purpose of the game is to develop a strategy to always win. As students continue to work on this they should become more interested in finding a strategy rather than winning. For students who have difficulty with this, decrease the number of markers used or only move one or two markers. Demonstrate how the markers can be grouped and ask students critical questions such as the importance of moving first, and other strategic moves. The game can also be made into a more difficult version to challenge higher ability students. Use two piles and change the rules. Students can take one marker from each pile or one marker from only one pile.	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Demonstrates a willingness to find a solution to a problem.	In order to develop the students' willingness to find solutions, the teacher should: a) create a positive classroom atmosphere that allows students to foster their own ideas and approaches in problem solving b) be supportive and encourage risk taking in finding solutions c) encourage students to use creative approaches d) be willing to accept unconventional solutions, more than one solution, or no solution (where appropriate) e) challenge students to think critically and justify strategies and solutions f) be enthusiastic and capable of recognizing the students' willingness and perseverance to solve problems g) provide appropriate questions for students h) present problem situations that enable students to gain problem-solving experience that is transferable to other subject areas and everyday life.	Students who experience difficulty with the complex strategies may find it necessary to use a more concrete approach for a longer period of time and may require more teacher guidance. e.g., A store owner buys candies in bulk bags containing 80 candies each. He re-packages the candies for sale in smaller bags of 12. How many candies are left over when one bulk bag of 80 candies is re-packaged?	Computers may be used to assist in teaching problem solving. Various programs and simulations require the use of particular or various strategies (e.g., Houghton Mifflin MAC, MECC, and Sunburst Communications Software). The use of relevant and realistic problems (from sources such as newspapers and magazines) is encouraged because this will increase the interest of students. Students may also be able to contribute their own ideas of problems to solve.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Demonstrates a willingness to find a solution to a problem. (cont'd)		Use of a concrete example will help students who experience difficulty with the operation of division. Concrete approaches should be encouraged as long as it is necessary. A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider the possibilities such as: a) other strategies and solutions b) "what if?" (change an element of the problem) c) generalization of rules to other situations. e.g., Using the above candy problem, ask: "How many bulk bags of 80 candies each would the store owner need to re-package so that no candies are left over?"	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Uses a variety of strategies to solve problems. Previously developed strategies are used. The following strategies should be developed throughout the various strands of the program and within the problem-solving framework: a) Understanding the problem • interprets pictures, charts and graphs • asks relevant questions	Students should encounter new situations that require an extension of problem-solving strategies acquired in Grade 7. The approach to this may be similar to that employed at the Grade 7 level whereby three non-related but similar problems can be chosen to focus on a particular strategy. As a teacher demonstration, involving the class, the strategy of working backwards can be developed within the problem-solving framework (Understanding the problem, Developing a plan, Carrying out the plan, Looking back) as follows: The number of a past year is divided by 2 and the result turned upside down and divided by 3, then left right side up and divided by 2. Then the digits in the result are reversed to make 13. What is the past year? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 42, #28.) Under the guidance of the teacher, students investigate the strategy by solving a similar but non-related problem such as: A boy attempts to climb a 10 m pole. At every attempt he climbs 1 m and slips back 1/2 m. After how many attempts will he have reached the top? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, p. 29, #8, 12.)	The Scope and Sequence in the Teacher's Edition (Holtmath 8) and Teacher Resource Manual (Journeys in Math 8) identify specific problem-solving strategies.	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. a) Understanding the problem (cont'd) - interprets pictures, charts and graphs - asks relevant questions	The natural progression leads to students using the same strategy to solve a problem such as: Janice went to a store, spent half of her money, and then spent $10 more. She went to a second store, spent half of her remaining money, and then spent $10 more. Then she had no money left. How much money did she have in the beginning when she went to the first store? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 10, #2.) Evaluation The evaluation of problem solving requires more than grading the solutions to mathematical problems. Continual observation and questioning of students while they are solving problems is essential. In assessing a student's problem-solving skills, the teacher should consider: - willingness to attempt problems - use of systematic approach - selection of appropriate strategies - efficiency in selection of appropriate strategies	A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider the possibilities such as: - other strategies and solutions - "what if?" (change an element of the problem) - generalization of rules to other situations.	Holtmath 8 Teacher's Edition, p. 83.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. b) Developing a plan (choosing a strategy) • collects and organizes information (charts and graphs) • makes diagrams and models • experiments through the use of manipulatives • breaks the problem into smaller parts • works backwards c) Carrying out the plan • applies selected strategies • presents ideas clearly • documents the process • works with care • works in a group situation d) Looking back • makes and solves similar problems.	• logical justification of strategies and solutions • perseverance • growth of confidence in problem-solving ability • transfer of problem-solving skills to situations other than mathematics. Evaluation techniques and instruments for problem solving are found in the Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, pp. 7, 8, 52-56. (See Holtmath 8 Teacher's Edition, p. iv Problem-Solving Strategies.)		e.g., Ann, Beth, Cathy, Dee, and Evita were playing in matches to see who would play position 1, 2, 3, 4, or 5 on the girls' tennis team. Each girl played each of the other girls once. How many matches were played? e.g., Children often use the *make-an-organized-list* strategy when solving this problem. First, a child might list all the matches that person A plays. Then the child would list all the matches played by persons B, C, D, and E. Note that once match AB is listed, match BA involves the same players and is not a different match. AB AC AD AE BC BD BE CD CE DE (Arithmetic Teacher, Vol. 32, #4, December 1984, p. 30.)

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.	The intent is to develop non-routine problem-solving skills in every student. Each student should understand the concepts of the problem-solving framework and develop multiple strategies.	Develop "Happy Numbers" (from Math Teacher, p. 618, November 1986). (R) Find and display all happy numbers less than 100. 19 is a happy number. Let's see why. 19 → 1 × 1 + 9 × 9 = 82 82 → 8 × 8 + 2 × 2 = 68 68 → 6 × 6 + 8 × 8 = 100 100 → 1 × 1 + 0 × 0 + 0 × 0 = 1 Since the sequence of numbers 19, 82, 68, 100, and 1 ends with the number 1, we say that the number 19 is a happy number. HAPPY NUMBERS 1 7 10 13 19 23 28 31 32 44 49 68 70 79 82 86 91 94 97	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations. (cont'd)		(E) Develop a program for "Happy Numbers", such as:	

```
 10  PRINT "HAPPY NUMBERS"
 20  PRINT
 30  REM CHECK ALL NATURAL
     NUMBERS LESS THAN 100
 40  FOR X = 1 TO 99
 50  Z = X
 60  REM GENERATE SIX TERMS
 70  FOR N = 1 TO 6
 80  IF Z > = 100 THEN 170
 90  IF Z > = 10 THEN 200
100  Z = Z * Z
110  NEXT N
120  REM CHECK FOR HAPPY
     NUMBERS
130  IF Z = 1 THEN PRINT X,
140  NEXT X
150  GOTO 220
160  REM ISOLATE THE DIGITS OF
     THE 3-DIGIT NUMBERS
170  H = INT (Z/100):T =
     INT (Z/10) - H * 10:U = Z -
     (H * 100 + T * 10)
180  Z = H * H + T * T + U * U:
     GOTO 110
190  REM ISOLATE THE DIGITS OF
     THE 2-DIGIT NUMBERS
200  T = INT (Z/10):U = Z - T * 10
210  Z = T * T + U * U: GOTO 110
220  END
```

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.	An equal emphasis should be placed on the various strategies for computing. Single-digit basic facts should be drilled on a regular basis through activities such as timed challenges or games. Paper-and-pencil strategies should be used to develop an understanding of sub-concepts such as re-grouping, borrowing or place value. Long and tiresome paper-and-pencil drill is discouraged. Estimation should be done on a daily basis. Recognition of appropriate situations for estimates, determining how precise an estimate should be for a given situation, and knowing when a computed answer is possible, are among skills to be emphasized. Mental computation involves using natural and easy strategies to compute exact answers. Strategies should be identified and shared as they evolve. Calculators should be used to develop understanding to investigate patterns, and to perform (tedious) computations that do not enhance understanding.		The following BASIC program should give your students an opportunity to practise their skills at estimation. The program can easily be modified to provide practice with any of the operations. It will work with both whole numbers and decimals. Each player takes turns inputting a value of their choice. Then each is asked to estimate the results of using a particular operation on the two numbers. The estimate closest to the actual answer is declared the winner. This program is written in a simple form of BASIC and should run on any computer that uses BASIC. (Arithmetic Teacher, Vol. 32, #5, January 1985.)

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations. (cont'd)			

```
  5  REM ESTIMATION GAME
 10  LET OP$ = "SUM"
 15  PRINT "IN TURN EACH PLAYER
     WILL ENTER"
 20  PRINT "A NUMBER OF THEIR
     CHOICE"
 25  PRINT
 30  PRINT "THEN EACH PLAYER
     WILL ENTER AN"
 35  PRINT "ESTIMATE OF THE
     ":OP$:" OF THE NUMBERS."
 40  PRINT
 45  PRINT "THE CLOSEST
     ESTIMATE WINS !"
 50  PRINT
 55  INPUT "FIRST PLAYER'S
     NUMBER = >":A
 60  INPUT "SECOND PLAYER'S
     NUMBER = >":B
 65  PRINT
 70  INPUT "FIRST PLAYER'S
     ESTIMATE. . .":A1
 75  PRINT
 80  INPUT "SECOND PLAYER'S
     ESTIMATE. . .":B1
 85  LET C = A + B:REM FINDS SUM
 90  IF C - A1 < C - B1 THEN 115
 95  PRINT
100  PRINT "PLAYER #2 WINS !"
105  GOTO 120
110  PRINT
115  PRINT "PLAYER #1 WINS !"
120  END
```

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations. (cont'd).			Line 10 can be changed to indicate a different operation. By changing line 10 to LET OP$ = "PRODUCT", the instructions in line 35 tell the user to estimate the answer to a multiplication problem. If line 10 is changed, then the operation in line 85 must also change. If OP$ = "PRODUCT", then line 85 must be LET C = A*B. The following lines can be added to make the program repeat. 120 PRINT 125 INPUT "AGAIN (Y/N)?": Y$ 130 IF Y$ < > "Y" THEN 200 135 GOTO 40 200 END

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A Whole Numbers			
1. Maintains previously developed skills with whole numbers (operations, order of operations, evaluation of expressions, prime numbers, factorization, divisibility)			
2 Finds the greatest common factor of a given set of numbers	Review Grade 7 notes on whole numbers Objectives #7, 8 and 10 as methods to determine the GCF. Use calculators and computers to determine the GCF of a set of numbers. Use GCF's to reduce fractions to lowest terms. Methods of finding GCF's include: i) listing the factors of the numbers, and finding the common factors Example: Factors of 24 = {①,② 3, ④ 6, ⑧, 12, 24} Factors of 32 = {①,②,④,⑧, 16, 32} GCF = 8	The listings on the right can be used to find GCF's on a computer. The first listing will determine the GCF of two numbers. Students could type the program onto the computer and use it. (E) An enrichment activity could be to have students either write their own program or improve the first listing. (E) Another enrichment activity could be to change the first listing so that the computer will determine the GCF of more than two numbers. Listing (b) is a modification of listing (a); the changes and additions will allow the computer to find the GCF of four numbers.	e.g., Darren wants to cut a 32 cm by 20 cm birthday cake into square pieces. What is the largest size he can cut? How many square pieces does he cut?

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A Whole Numbers 2 Finds the greatest common factor of a given set of numbers (cont'd)	ii) finding the prime factors of the numbers and then determining which factors are common Example: $24 = 2 \times 2 \times 2 \times 3$ $32 = 2 \times 2 \times 2 \times 2 \times 2$ $GCF = 2 \times 2 \times 2 = 8$ Simplify: $\frac{24}{32}$ $\frac{24 \div 8}{32 \div 8} = \frac{3}{4}$ (simplest terms)		BASIC Listing:

```
  5  REM GCF FOR 2 NUMBERS
 10  HOME
 20  INPUT "FIRST NUMBER?";A
 30  INPUT "SECOND NUMBER?";B
 40  IF A>B THEN N = B:GOTO 60
 50  N = A
 60  FOR X = 1 TO N
 70  Y = A/X : Z = B/X
 80  IF Y = INT(Y) AND Z = INT(Z)
     THEN CF = X
 90  NEXT
100  PRINT:PRINT
110  PRINT "THE GREATEST
     COMMON FACTOR IS";CF
120  END
```

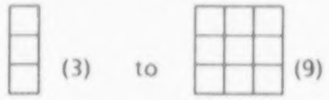
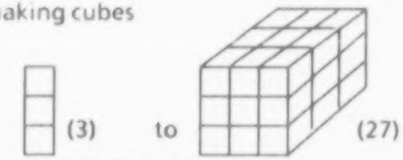
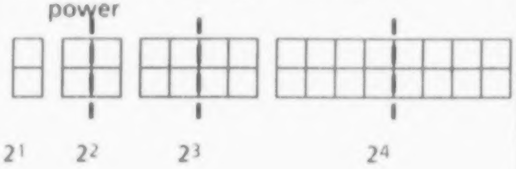
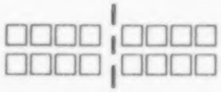
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 2. Finds the greatest common factor of a given set of numbers. (cont'd)			(Holtmath 7 Teacher's Edition - GCF/LCM, p. 144-145.)

```
  5  REM GCF FOR 4 NUMBERS
 10  HOME
 20  INPUT "FIRST NUMBER?";A
 22  N = A
 25  INPUT "SECOND NUMBER?";B
 28  IF A>B THEN N = B
 30  INPUT "THIRD NUMBER?";C
 32  IF C<N THEN N = C
 35  INPUT "FOURTH NUMBER?";D
 40  IF D<N THEN N = D
 50  FOR X  =  1 TO N
 70  Y1 = A/X:Y2 = B/X:Y3 = C/X:
     Y4 = D/X
 80  IF Y1 = INT (Y1) AND
     Y2 = INT(Y2) AND Y3 = INT(Y3)
     AND Y4 = INT(Y4) THEN CF = X
 90  NEXT
100  PRINT:PRINT
110  PRINT "THE GREATEST
     COMMON FACTOR IS":CF
120  END
```

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 3. Finds the lowest common multiple of a given set of numbers.	Apply concepts from Grade 7 Whole Numbers Objectives #7, 8, and 10 as strategies to attain the LCM. Try a guess and check strategy to find LCM. Relate LCM to lowest common denominator to assist in making equivalent fractions for operational purposes. Stress the meanings rather than the acronym (GCF, LCM).	(E) Program a computer to select the LCM of given numbers. (See previous objective.) (E) The listing at right could be improved or could be changed to find the LCM of more than two numbers (as was done for GCF's). (E) Utilize the relationship between the GCF and LCM of two numbers (A and B): GCF × LCM = A × B.	BASIC Listings:

```
  5  REM LCM OF 2 NUMBERS
 10  HOME
 20  INPUT "FIRST NUMBER?";A
 30  INPUT "SECOND NUMBER?";B
 40  IF A<B THEN
     N = B:P = A:GOTO 60
 50  N = A: P = B
 60  X = 0
 70  X = X + 1
 80  Y = N * X/P
 90  IF Y = INT(Y) THEN LCM = N*X:
     GOTO 110
100  GOTO 70
110  PRINT:PRINT
120  PRINT "THE LOWEST
     COMMON MULTIPLE IS"; LCM
130  END
```

Software: Holtmath 7 LCM, pp. 146-147.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 4. Understands and uses the terms exponent, base, power, squared and cubed and the nth power of a number	Write the value of a power with a whole number base and exponent (see Grade 7 notes on Whole Numbers Objective #4) a) Use blocks as manipulatives to develop concepts of making squares  or making cubes  See Holtmath 7 Teacher's Edition, p. 26 b) Use paper folding to develop the nth power  OR If 2^2 = and 2^n =  then discuss the value of *n*.	(E) Continue development to encompass square roots using blocks. Have students construct squares; use area and dimensions to develop an understanding of a square root. (E) Develop law of multiplication with exponents.	Using grid paper on the overhead ask students to use their calculators to guess and check and find *n*. e.g., if 2*n* = 1064 *n* = ? Use calculators in a 'mad minute' activity to reinforce concept.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A. Whole Numbers 5. Demonstrates the need for scientific notation.	(See Holtmath 8 Teacher's Edition, pp. 56-57. Journeys in Math 8 TRM, p. 71.)	(R) Round off numbers to approximate answers. Multiply whole numbers and count zeros to avoid writing all of the zeros.	Use large numbers and calculators to emphasize the need for scientific notation. e.g., 87000 × 5670 = n a) Discuss calculator output such as: • 49 329 000 E where multiplication result is too large for calculator to handle • 4.9329 8 represents scientific notation as 4.9329 × 10^8 b) Use computer to type the following command – PRINT 670 924 000 – discuss its output of 6.70 924E + 08

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
A Whole Numbers 6 Writes numbers in scientific notation, and scientific notation numbers in standard form (limit: positive exponents).	(See Holtmath 8 Teacher's Edition, pp. 56-57. Journeys in Math 8 TRM, p. 71.)	Students work in pairs to make and use flashcards to convert numbers from scientific notation to standard form and vice versa. e.g., 6×10^8 FRONT 600 000 000 BACK	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
B. Integers 1. Maintains previously developed skills with integers (need for integers, concept of integers, ordering of integers, demonstrates addition of integers with manipulatives).	See Grade 7 notes.		
2. States the additive inverse of any integer.	Use the terms "additive inverse" and "opposite" interchangeably. Discuss temperature, bank account and above/below sea level to develop the concept of additive inverse/opposite. Students should understand that the additive inverse is a tool for computation. (Number + Additive Inverse = 0) ("Concrete Development" Journeys in Math 7 TRM, p. 302.)	(R) Use a strip of paper to develop a number line. e.g., −4 −3 −2 −1 0 1 2 3 4 ↑ Fold on zero, and find the additive inverse.	Holtmath 7 Courseware, Disk 5, Additive Inverse.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
B. Integers 3. Uses concrete manipulatives to demonstrate the subtraction, multiplication and division of integers.	Concrete development (subtraction). (Journeys in Math 7 TRM, pp. 304-305.) Extend the idea of changes and coloured chips to multiplication and division. Multiplication and Division Emphasize the meaning of the negative sign (opposite). Give each student a red (–) and a black (+) chip. Ask the students to replace the black chip with the opposite colour (one negative sign). What would happen if they were asked to replace the chip with the opposite colour two times (two negative signs)? Three times? Four times? The notion that pairs of opposites (negative signs) cancel each other should immerge. Begin by using the black chips (+) to demonstrate multiplication as repeated addition and division as repeated subtraction. The colour of the chips to be used in the demonstration will change as integral values are introduced and will be determined by the number of negatives (opposites) in the question (e.g., 2 X $^{-}3$ means one opposite, or that red chips will be used in the demonstration). **Note**: Students must write a number sentence to describe each operation or activity.		Holtmath 7 Teacher's Edition, pp. 274-275.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
B. Integers 4. Performs the operations of addition, subtraction, multiplication and division with integers using paper-and-pencil algorithms, estimation, mental computation and a calculator.	Concrete activities should be extended to determine strategies for computing without manipulatives. As students write number sentences describing concrete operations, they should be looking for patterns that would enable them to perform the operations without the manipulatives. An equal emphasis should be placed on mental computation, pencil-and-paper computation, estimation and using the calculator. Timed activities and games may be used to encourage mental facility.	(E) Journeys in Math 7 Teaching Aids. Games: 21 (p. 81) 23 (p. 82) 24 (p. 82)	Students should be taught how to compute with integers on a calculator. Because calculators may vary in their operating systems, some time should be taken to investigate the change sign key and to verify the operating system of the calculator. Students should be encouraged to verbalize how their own calculators work when operating with integers. Additional: See Journeys in Math 7 TRM, p. 309.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
C. Rational Numbers			
1. Maintains previously developed skills with decimal numbers (place value, operations, ordering, rounding, order of operations).			
2. Maintains previously developed skills with fractional numbers (concept of a fraction, equivalent fraction, basic fraction, mixed numbers, improper fraction, ordering fractions, concrete operations with fractions, order of operations).			
3. Writes the multiplicative inverse (reciprocal) of a fraction, whole number or integer.	Develop reciprocals for proper fractions only. Students should understand how the reciprocal/multiplicative inverse acts as a tool (number × reciprocal = 1). Get students to realize that dividing by 2 is the same as taking one half of an item. Progress to show ÷ 3 = × 1/3, ÷ 7 = × 1/7. Discuss the relationships between 2 with 1/2, 3 with 1/3, and 7 with 1/7. Extend to what happens to 2/3.		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
C. Rational Numbers 4. Performs the operations of addition, subtraction, multiplication and division with fractions (limit: positive rationals).	Review the four operations with fractions at a concrete level. Students should be required to write number sentences to describe the concrete operations while looking for patterns that would enable them to perform the operations without using manipulatives. The fractions used to perform initial paper-and-pencil or mental operations should be relatively simple and should easily be related to a concrete manipulative (especially if difficulties occur). Estimation of answers to computations with fractions should be encouraged continuously.	(R) Students should continue to use manipulatives or pictorial presentation until they are able to perform operations formally. (R) Encourage the students to make and use fraction slide rules. (E) Develop games like fraction dominos. (E) Extend strategies developed to include mixed numbers.	Holtmath 7, Disk 3
5. Demonstrates the need for rational numbers (e.g., $-3 \div 2 = ?$ No answer is possible without a set of rational numbers)	Discuss examples of uses of rational numbers: e.g., Measurement Banking – deposits, withdrawals, overdrafts Business – profits, losses		

NUMBER SYSTEMS AND OPERATIONS

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
C. Rational Numbers 6. Recognizes rational numbers as all numbers that can be written in the form: $\frac{a}{b}$ where $b \neq 0$.	It is important that students understand that numbers have many different but equivalent forms. Have the students construct a fraction tape (a strip of paper folded in halves, quarters, eighths; thirds, sixths, etc.) and mark the folds as one would mark a number line (zero in the middle, with rational numbers describing the folds on each side). Then ask the students to write the decimal and, if possible, integral equivalents to the rational numbers. Discuss the fact that any number (regardless of its form) that can be written as a fraction or a/b where $b \neq 0$ form, is by definition a rational number. Use a calculator to explore the various patterns formed by fractions and their equivalent decimal forms (e.g., 1/7, 2/7, etc., 1/9, 2/9, etc., 1/11, 2/11, etc.).	(E) Are there numbers that do not have fractional equivalents? What are these numbers (irrational numbers)? How are they generated? (See Journeys in Math 8 TRM, p. 157.)	Journeys in Math 8 TRM, p. 157, "Calculator for Rational and Irrational Numbers".
7. Compares and orders rational numbers using <, > or =.	Use the fraction tape (Objective #6) to compare and order rational numbers. It should be made clear to students that on a number line, numbers on the right are larger than numbers on the left (e.g., $1 > {}^{-}3$).	(R) Use calculator to make an equivalent form to explore differences. (E) See Journeys in Math 7 TRM, pp. 174 – 175.	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
C Rational Numbers 8 Uses a number line to demonstrate the relationship between whole numbers, integers, fractions and rationals.	Extend the fraction tape. Discuss the relationship between whole numbers, integers, fractions and rationals.		

RATIO AND PROPORTION

Grade 8

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.	The intent is to develop non-routine problem-solving skills in every student. Each student should understand the concepts of the problem-solving framework and develop multiple strategies. (Refer to Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985.)		
2. Maintains previously developed skills (understands and constructs ratios, equivalent ratios; finds missing element of a proportion, percent as a ratio, percents as decimals, percents of numbers; and expresses one number as a percent of another).	Encourage the development of verbal activities (mental facility) such as "mad minute" to reinforce maintenance skills. e.g., (i) "Give two equivalent ratios for..." (ii) "Express this ratio in three ways..." (iii) Find the unknown 3/4 = x/20 using two methods. (iv) Advertisements which illustrate percentage discounts. (v) Estimate percentages of test scores and check using a calculator. Students use software from Mac Courseware 8 – Disk B program, 9B – Ratio Rendezvous. This will help students to develop their own method of identifying equivalent ratios.	(E) It is important to clarify that fractions are ratios but ratios are not fractions! e.g., 400/700 ⇄ 4/7 ratio 400/700 = 4/7 fraction Note: 1girl/2 boys Ratio 1 as to 2 Number of girls not 1/2 (R) For concrete development of these skills see detailed explanation at the Grade 7 level.	MAC Courseware 9A Disk B. Ratio Rendezvous 9B. (This should be used for enrichment.)

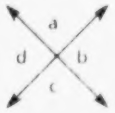
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Maintains previously developed skills (understands and constructs ratios, equivalent ratios; finds missing element of a proportion, percent as a ratio, percents as decimals, percents of numbers; and expresses one number as a percent of another). (cont'd)	A practical example is mixing photography chemicals, e.g., a) 1 part of Water : 9 parts of Developer b) 2 parts of W : 18 parts of D c) 3 parts of W : ? parts of D Students can investigate other examples $\frac{1}{9} = \frac{2}{18}$ Students should be encouraged to suggest other practical examples.	(E) Students can investigate trends in the stock market. If a written report is required, the criteria for evaluation must be clearly stated.	
3. Gives examples of ratios involving situations where the equivalent percent is greater than 100.	Discuss what a percent greater than 100 means and its practical occurrences (stock markets, growth in industry, retail sales). a) Use 10 × 10 grids to demonstrate percents greater than 100. Have students shade the grids to show, for example, 120% (one complete 10 × 10 grid and 20 squares from a second grid). Note that 120% is a number larger than 1 but smaller than 2.		

<table>
<tr><th>Objective</th><th>Clarification or Example</th><th>Elective Suggestions</th><th>Integration of Problem Solving and Technology</th></tr>
<tr><td>3. Gives examples of ratios involving situations where the equivalent percent is greater than 100. (cont'd)</td><td>b) Use a chart similar to the one below.
<table>
<tr><th>Cost Price</th><th>Selling Price</th><th>Profit</th><th>% Profit (x)</th></tr>
<tr><td>$20</td><td>$25</td><td>$5</td><td>$\frac{5}{20} = \frac{x}{100}$ ∴x = 25%</td></tr>
<tr><td>$20</td><td>$30</td><td>$10</td><td>$\frac{10}{20} = \frac{x}{100}$ ∴x = 50%</td></tr>
<tr><td>$20</td><td>$40</td><td>$20</td><td>$\frac{20}{20} = \frac{x}{100}$ ∴x = 100%</td></tr>
<tr><td>$20</td><td>$60</td><td>$60</td><td>$\frac{40}{20} = \frac{x}{100}$ ∴x = 200%</td></tr>
</table>
Let students suggest other examples.</td><td></td><td></td></tr>
<tr><td>4. Converts mixed numbers to percents and vice versa.</td><td>Percents larger than 100 have equivalent mixed number forms. Extend the development from Objective 3 to include many examples of mixed numbers (more than one 10 × 10 grid) and their equivalent percents.</td><td></td><td>Understanding the operation is a necessary requirement before using a calculator or computer. Before permitting (or instructing) students on using the percent key on the calculator, ask the students to describe (in a paragraph) how the percent key on their own calculator works. In other words, describe the algorithm that the engineer (builder) used to "program" the calculator.</td></tr>
</table>

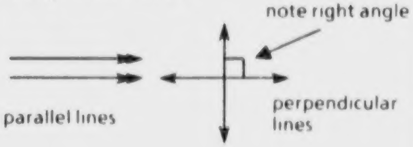
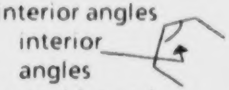
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5 Given the percent determines the missing value in applications such as discounts, increases, decreases, or sales tax.	Groups of students research discounts, increases, decreases or sales tax using newspaper advertisements. Classroom discussion can follow, stressing mental computation and estimation. Use the calculator to check, e.g., (best deal - discount). Ask students to collect daily newspaper articles. Discuss the meanings of: 50% more, 30% discount, 5% sales tax. Follow up with students creating their own advertisements.		Use the calculator to check real examples of discounts collected from local newspapers. A chart may be used to organize the information. Item: Suit Reg. Price: $150 % Discount: 30% i.e., 30 ÷ 100 $\boxed{\%}$ Amount of Reduction: 0.3 × 150 = 45 Sale Price: 150 − 45 = 105 Item: Stereo Reg. Price: $280 % Discount: 15% i.e., 15 ÷ 100 $\boxed{\%}$ Amount of Reduction: 0.15 × 280 = 42 Sale Price: 280 − 42 = 238 Note: $\boxed{\%}$ Represents the percent key on calculators.

RATIO AND PROPORTION

<table>
<tr><th>Objective</th><th>Clarification or Example</th><th>Elective Suggestions</th><th>Integration of Problem Solving and Technology</th></tr>
<tr><td>6. Understands and writes rates as the comparison of two numbers with different units (e.g., 15 km/2h or 3 items/$1).</td><td>(Holtmath 8 Teacher's Edition, pp. 140-141. Journeys in Math 8 TRM, p. 178.)</td><td>(E) Use statistics in sports to calculate the m.v.p. (most valuable player) for a particular team or sport.
Batting Statistics
<table><tr><th></th><th>At Bat</th><th>Hits</th></tr><tr><td>Tom</td><td>60</td><td>10</td></tr><tr><td>Dick</td><td>40</td><td>8</td></tr><tr><td>Harry</td><td>50</td><td>20</td></tr></table>
(See Journeys in Math 7 TRM, p. 219.)</td><td></td></tr>
<tr><td>7. Writes proportions involving rates.</td><td>(Holtmath 8 Teacher's Edition, p. 136 "Alternative Teaching Strategies". Journeys in Math 8 TRM, p. 170.)</td><td></td><td></td></tr>
<tr><td>8. Finds the missing element in a proportion involving rates.</td><td>(Holtmath 8 Teacher's Edition, pp. 136-139. Journeys in Math 8 TRM, p. 172.)</td><td>(E) Students compare sizes of a selected product to determine which is the most economical to buy. The intent is for students to find the unit rates (comparison shopping).
(Holtmath 8 Teacher's Edition, pp. 141-145.)</td><td></td></tr>
</table>

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.			
2. Maintains previously developed skills (linear, area, volume, capacity and mass units of measure; uses geometric tools to measure line segments and angles and to construct geometric designs; transformational geometry).			
3. Understands and uses the terms perpendicular and parallel lines.	(Holtmath 8 Teacher's Edition, pp. 178-181. Journeys in Math 8 TRM, p. 234.)	(E) Explore the relationships between angles of intersecting lines. e.g.,  $a = c, d = b, b + c = 180°$	

217 218

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Understands and uses the terms perpendicular and parallel lines. (cont'd)		Give pictorial diagrams to show difference between perpendicular and parallel lines. 	
4. Draws or sketches various polygons using tools such as a computer, compass, straightedge, ruler, protractor.	The intent of this objective is to investigate the attributes (characteristics) of polygons which will lead to a definition of a polygon (a plane closed figure whose sides are line segments). Further "investigation" of the attributes (e.g., measures of sides, congruence of sides and/or angles, measure of interior angles, number or length of diagonals) can lead to the classification of the polygons (according to number of sides or as regular) which is objective #5. Students should be asked to place a number of dots (points) randomly on a paper (plane). Keep the number of points small (2-10). Then ask students to connect the points to form various figures. Discuss the outcome by grouping (classifying) the figures using the following points as a guide:	The following points may be used as a guide in the investigations and subsequent discussions: a) congruent sides b) congruent interior angles  c) both congruent sides and angles are conditions for a regular polygon d) number of diagonals e) relationship of the perimeter of a regular polygon to the length of its diagonal.	The following LOGO procedure is an example of a program that may be generated by students. This procedure will allow the construction of various polygons by "inputting" the length of a side, and the measure of the interior angle. For example, SHAPE: 40, 60 will result in a square being plotted on the screen. Students should be encouraged to change the measures of the angles (and the sides, in cases where the shape is too small or too large for the screen) to construct varying shapes or polygons. It must be noted that this procedure will not stop unless it is interrupted (with a CONTROL-G command in the case of Apple LOGO). The results should be tabulated: length of side; size of

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Draws or sketches various polygons using tools such as a computer, compass, straightedge, ruler, protractor. (cont'd)	a) the figure must be closed (have an interior and an exterior) b) line segments (not curved lines) define the sides of a polygon c) the minimum number of points required to close a figure is 3. Further investigation will lead to the classification of the polygons according to the number of sides. Many techniques or instruments may be used to perform the actual sketches or constructions. Traditional tools such as the compass, protractor or ruler have "built-in" mathematics concepts but their use may be tedious. The computer will allow students (after learning to use a graphics program or a program such as LOGO) to generate many and varying shapes in a short period of time or to "instruct" a computer to construct a polygon. (For more information on using a LOGO program see Journeys in Math 8 TRM, pp. 91, 94-95.)		angle (note that the angle is not the interior angle, but the supplement of the interior angle (180 – ANGLE) in this procedure); and the shape of the figure (polygon or start). LOGO Listing TO SHAPE: SIDE ANGLE FD: SIDE RT: ANGLE SHAPE: SIDE: ANGLE END

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Identifies and classifies polygons according to the number of sides (limit: decagon).	See Objective #4.	(E) Students could investigate to find the names of some polygons with more than 10 sides.	
6. Investigates triangles by examining attributes such as measure of angles, measure of sides and lines of symmetry.	Investigate triangles by having students construct a number of random triangles. Discuss their similarities and differences and the different attributes (length of sides, measure of angles, number of lines of symmetry) that may be used to classify the triangles (before assigning names to the classifications). (See Holtmath 8 Teacher's Edition, pp. 184-185. Journeys in Math 8 TRM, pp. 242-243.)	(E) See Holtmath 8 Teacher's Edition, pp. 186-187, "Rigid and Non-Rigid Shapes".	
7. Investigates quadrilaterals by examining attributes such as measure of sides, measure of angles, lines of symmetry and diagonals.	A discovery approach is encouraged. Either provide to students, or have them construct (from black line masters) a "quadrilateral kit". This kit should contain a number of varying shapes that are large enough to measure and manipulate: irregular quadrilaterals, trapezoids, parallelograms, rhombi, rectangles and squares. Discuss the various attributes of the shapes and then have the students group the shapes according to those attributes. Note that some of the quadrilaterals can fit into several categories.	(E) Journeys in Math 7 TRM, p. 145.	Problem Solving: Explore tangram-type puzzles. Computer: Holtmath 7, Software Disk 4, "Angles of Regular Polygons".

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
7. Investigates quadrilaterals by examining attributes such as measure of sides, measure of angles, lines of symmetry and diagonals. (cont'd)	e.g., Quadrilaterals that have: four sides; (at least) one pair of parallel sides; opposite parallel sides; opposite sides are congruent (Can opposite sides of quadrilaterals be parallel without being congruent?); congruent opposite angles; opposite parallel sides and at least one right angle; number of lines of symmetry; number of diagonals; congruent diagonals; diagonals bisect each other; diagonals meet at right angles. Develop a classification scheme based on the properties of the quadrilaterals, which would relate quadrilaterals, trapezoids, parallelograms, rhombi and squares. Alternative developments: (Journeys in Math 7 TRM, pp. 144-145. Journeys in Math 8 TRM, pp. 244-245. Holtmath 8 Teacher's Edition, pp. 194-195.)		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
8. Adds, subtracts, multiplies and divides using SI units of measure.	Only commonly used units (km, m, cm, and mm) should be used to develop this objective. The operations with units of measure should be developed in the context of real problems (e.g., perimeter and area). To add, subtract, multiply or divide measures, the units must be common (the same) or else the result will not have a standard meaning (e.g., 3 m + 4 cm = ? or 2 cm x 5 mm = ?). Multiplication (or its inverse-division) of measures is a difficult concept. Students should be taught that whenever they see, for example, cm x cm this does not mean cm "times" cm but, rather, is an expression of area (a count of square centimetres (cm^2)). This objective may be developed within the context of objectives #9 (perimeter) and #10 (area).	(E) Explore the relationship between, for example, 1m^2 and cm^2. (E) Explore the unit of area hectare (ha).	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
9. Understands and uses formulas as indirect measures of the perimeter of polygons (includes regular polygons).	Formulas are indirect strategies for calculating measures that can sometimes be directly measured. Students should first have experiences with the direct measures before attempting to devise a strategy (or formula) for computing the measure. For perimeter this experience may be using a string (and a ruler) to determine the distance around different polygons; then using only a ruler to measure the sides and then determining the sum of the sides. Students should note that the order in which the measures are taken or summed, does not affect the result. Students should verbalize a strategy before attempting to generalize it in the form of a formula. For example, "the perimeter of a rectangle is two lengths added to two widths because . . ." The generalization or the formula should be the last stage of the development and may be used to expedite the process of determining a perimeter. The use of formulas may not be appropriate for all students.	(R/E) Journeys in Math 8 TRM, pp. 80-81.	Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985. p. 47, #6.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
10 Understands and uses formulas as indirect measures of the area of polygons (triangles, all parallelograms and trapezoids)	Formulas are indirect strategies for calculating measures that can sometimes be directly measured. Students should first have experiences with the direct measures before attempting to devise a strategy (or formula) for computing the measure. For area, this experience is counting the number of squares that can be found within a closed region. Transparency grids (cm x cm) may be used to determine the areas of both regular- and irregular- shaped objects. The measures should then move on to rectangles where it should be discovered that, the number of squares in one row corresponds to the length, and the number of rows corresponds to the width. Hence, the product of the length and the width determines area. Students should be encouraged to verbalize the strategy before generalizing it. Areas of other polygons (e.g., triangle, parallelogram) are variations of the rectangle. (See Holtmath 8 Teacher's Edition, pp. 74-82. Journeys in Math 8 TRM, pp. 83-87.)		Computer: Mathematics Activities Courseware 8, "Metric Mysteries", Disk C, Houghton Mifflin.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
11. Performs an experiment to determine the value of π and understands π as a ratio of the circumference of a circle divided by its diameter. (i.e., $\pi = \frac{C}{d}$)	An introductory activity for this objective could be to draw different sizes of a particular polygon and then determine the ratio of the perimeter to the length of a diagonal (have the students make the measurements). e.g., For a set of squares, $\frac{\text{perimeter}}{\text{diagonal}} = 2.9$ From this activity the students should find the principle that, if figures have the same shape, the ratios of the corresponding parts remain constant even though the size is different. Using the above activity the students might predict that $\frac{C}{d}$ will be a constant (because all circles have the same shape). By measuring various round objects (records, coins, jars, cans) using a string and a ruler (a metre stick, if necessary) and also keeping a record of the diameters and circumferences, the concept that π is a ratio of C to d should be discovered.	(E) Use the library or other resources to gather information about the historical development of π. (E) Develop a strategy for finding the diameter of a circle when the centre is not given. (E) Journeys in Math 8 TRM, p. 82.	Problem Solving: By comparing measurements, the question of accuracy should arise. When is it important? When is an estimation appropriate?

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
12. Understands and uses the formula $C = \pi d$ as an indirect measure of the circumference of a circle.	Using the measurements from Objective #11, compare calculated circumference ($C = \pi d$) to measured circumference. The development of the formula as a strategy should flow directly from the previous topic. Given the radius, explore how to find the circumference. Two possibilities can be used $C = \pi d$ (double the radius and then utilize the formula), or $C = 2\pi r$.		Computer: Holtmath 7 Software, Disk 1, "Circumference of Circles".
13. Uses the formula $A = \pi r^2$ to indirectly determine the area of a circle given its radius or diameter.	Use a compass and grid paper and have students draw a circle with a specified radius. By counting squares, estimate the area. Compare estimates.	(E) Students show development of the formula $A = \pi r^2$. (E) Show that the area of a regular polygon ($A = \frac{1}{2}ans$ where a is the apothem, n is number of sides, and s is the measure of one side) approaches the area of a circle ($A = \pi r^2$) as the number of sides increases.	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
14. Draws or sketches a right rectangular prism.	From everyday objects such as a cereal box, or a kleenex box, discuss the relationship between sides. From this, students should be able to determine the characteristics of all right rectangular prisms. Prisms are easily drawn by using isometric paper or grid paper. (Journeys in Math 8 TRM, p. 98.)	(E) Journeys in Math 8 TRM, p. 98	
15. Understands and uses a formula as an indirect strategy for determining the volume of a right rectangular prism or a cube.	A formula is an indirect strategy for calculating a measure that can sometimes be directly obtained. Students should have experiences with obtaining the measure directly before determining a strategy for computing the measure. Use centimetre cubes (sugar cubes will do if other manipulatives are unavailable) to determine the volume of rectangular prisms. Note that the number of cubes in one layer corresponds to the area of the base (length times width) and that the number of layers corresponds to the height. (See Holtmath 8 Teacher's Edition, pp. 91-93. Journeys in Math 8 TRM, p. 102.)		

DATA MANAGEMENT **Grade 8**

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.	See Grade 7 Data Management Objective #1.		See Grade 7 Data Management Objective #1.
2. Maintains previously developed skills (understands the purpose of statistics; interprets data from tables and graphs; draws graphs). Extended Content Utilize graphs from newspapers to display a bulletin board.	a) Ask how numbers affect our lives. Use sources such as measurement, experimentation, observation, encyclopedias, newspapers, information services. b) Recap frequency tables, pictographs, bar graphs, line graphs, and circle graphs as ways of organizing and comparing data.	(R) Explain strengths and weaknesses of each graph. (E) Use histograms, stem-leaf tables, box and whisker plots.	Computer programs can display graphs. Houghton Mifflin, MAC 7, "Pie Graphs".

2[illegible] 240

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3 Understands and uses the terms bias, sample and population.	Demonstrate sampling by taking a survey of students (e.g., from one row in a class) who are wearing running shoes, and then compare the sample to the population of the class. Discuss the bias by asking if this sample is likely to be representative of the school population (including the teachers) and of the town or municipality. Discuss how to obtain a useful sample in each case. The same type of activity could be done to compare the number of people who wear glasses to those who do not. (See Journeys in Math 8 TRM, pp. 324-325.)	(R) Direct 3 or 4 pairs of students to complete a poll based on 10 responses on whether you like orange juice or apple juice. Ask students why results may vary. (E) Get students to conduct polls, interpret the results, and discuss problems that could arise.	Problem Solving: How are the fish in a lake counted?

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Distinguishes between a survey and a census, understands when each is used and potential biases that may occur (survey).	A survey is a random poll of an identified group for the purposes of collecting data or acquiring information about some aspect of the group or area. A survey may be biased if the sample was not random (i.e., if the sample was not representative of the population) or if the sample included a population outside the identified group. For example, a survey to determine the most popular television program among junior high students would be biased if only one class of Grade 7 students were polled (only one age group was represented in the sample) or if some elementary or high school students were included in the sample. A census is a count or poll of an entire population and provides accurate information. A census is not always practical because of the size of the identified group and the time and costs associated with the gathering of information. Discuss the merits of a survey and a census and the situations or conditions that would be required to make a census or survey appropriate. When is accuracy more important (population of a school to determine per pupil grants) and when is knowing the information quickly more	(R) Select three students to be a class representative. Choose one row to see who they would elect. Choose another row. Predict how the whole class would vote. Compare results to voting of entire class. (E) Determine methods of taking a survey typical: (young vs old; male vs female; regional locations; high income vs low income).	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Distinguishes between a survey and a census, understands when each is used and potential biases that may occur (survey). (cont'd)	important (deciding how many hot dogs and how many hamburgers should be purchased for a school function)?		
Extended Content a) How does one conduct a survey: personal interviews? questionnaires? telephone calls? interviews? b) Discuss whether answers to surveys should be restricted or unlimited. c) Discuss problems associated with census.	A discussion on how to ask questions could lead to realization that questions can influence the outcome of a survey? i.e., Do you feel the current tax hike is too high? What is your feeling on the proposed tax hike?		A data base like the Appleworks program can be used to sort information and/or 'pull out' the required information.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Recognizes the use and misuse of statistics in society (news reporting, census, polls, etc.).	Statistics are used to predict outcomes but one questions their reliability as one finds incomplete, misleading, and misrepresented data. Find examples of how one uses statistics to one's advantage. e.g., Political survey polls which predict party support are often based on a limited sample which could create varying outcomes.	(R) Predicting the standing of a sports team (baseball, hockey, local team) based on its record in the last ten (x) games may be misleading. A team with a good record in the last ten (x) games may still end up last in the standings over the whole season. (E) Watch a newscast on TV to determine where statistics are used, how, why, and discuss their accuracy and timelines, etc.	

247

248

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.			
2. Maintains previously developed skills (variable, evaluation of expressions, concept of equality, plots on a coordinate plane).			
3. Identifies and combines like terms.	(See Journeys in Math 8, pp. 346-347.)	(E) Use formal approach to explain answers to $x + 3x$, $7n - 3n$, $8y - 9y$. Expand into fractional variables.	
4. Uses formal procedures to solve equations of the form $x + a = b$, $ax = b$, $ax + b = c$, and $\frac{x}{a} = \frac{b}{c}$ (limit: positive rational numbers and integers).	Review the concept of equality. (See Objective #6, Algebra Grade 7.) Formal algebraic techniques may be introduced by using the concept of balance and so a balance scale lends itself here. For example, a nickel and 2 one-gram masses balance with 7 one-gram masses. The mass of the nickel (represented by x) can be found by removing 2 one-gram masses from each side (the opposite operation).	(E) Houghton Mifflin Mathematics Activities Courseware 8, Eckses and Ohzs, pp. 37-40.	Problem-solving strategies: experiment through the use of manipulatives; work backwards; break the problem into smaller parts; make and solve similar problems.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Verifies solutions to the equations.	Review the concept of evaluating expressions. Verifying a solution to an equation is simply a matter of using the solution independently to evaluate the expression on each side of the equation to determine if in fact the LHS (left-hand side) is equal to the RHS (right-hand side).	(E) Estimate solutions by guess and check strategy.	
6. Uses substitution and equation-solving techniques to find a missing element of a formula: e.g., If $p = 2$ and $q = 0.5$ find c in $p = \frac{c}{q}$.	The intent of this objective is to de-mystify formulas. The few formulas presented in mathematics classes are usually taught as a means to an end (e.g., $d = rt$ is used to find distance, rate or time). The result is that little transfer occurs in knowing how to use other formulas in mathematics and other disciplines. A much more holistic view of formulas must be taken. The outcome of this objective should be that students will understand relationships among the variables, recognize the similarities (or differences) among those relationships (regardless of what kind of variables are used), and will know how to substitute and solve for missing elements.	(E) Have students write their own nonsensical formulas and a funny story about what the variables represent. Exchange the formulae and stories within the class. For example: $E = 3 + 7a$ where E represents the number of elephants and *a* is the number of people with blue eyes. (E) Discuss the effect on a selected element of a formula if another element is doubled, tripled, etc. (R) Use real life situations where 3 apples and 4 oranges cost a certain amount of money. Give variations for 2 of the elements so the third element needs to be solved.	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
7. Generates a set of ordered pairs in a linear relation.	(See Holtmath 8 Teacher's Edition, pp. 290-295. Journeys in Math 8 TRM, pp. 308-309.)	(E) Given the ordered pairs, determine the relation. (R) Give values for x as 0, 1, and 2 or numbers that are conducive to easy solution. (E) Develop a computer program to generate ordered pairs.	(R) Use a computer to give the pairs.

```
  5 REM ordered pairs
 10 HOME
 20 PRINT "This program will
    generate ordered pairs for
    relation rules in the form
    ax + b."
 30 PRINT:PRINT "You will be
    asked to give values of a and b,
    and then the computer will
    print values for ax + b by using
    a range of -10 to + 10 for x."
 40 INPUT "What value of a?";A
 50 INPUT "What value of b?";B
 60 HOME
 70 PRINT "X", A; "X + ";B
 80 PRINT "_ _ _ _ _ _ _ _ _ _ _ _"
 90 FOR X = -10 TO 10
100 PRINT X, A*X + B
110 NEXT
120 END
```

253

254

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
8. Given a linear relation, constructs a table of values and a graph for that relation.	Give an example of a linear relation such as Journeys in Math 8 TRM, p. 310. Construct a table of values, generate the ordered pairs, and graph the relation. At this time discuss the merits of a graph as opposed to a table of values. e.g., – visible and more understand-able representation of a relationship. – can be used to predict or hypothesize. Given a linear relation of the form $y = ax + b$, construct a table of values. Using substitution with given *x* values, students find *y*. Extending this, students generate their own values for *x* to find *y*. From a table of values students try to determine the defining relation by looking at the pattern. Explore number patterns that arise from tables of values. This may open discussion about choosing *x* values, choice of scale, and extension to more than one quadrant.	(E) Given a linear relation of the form $y = ax + b$, explore how changing one aspect (*a* or *b*) of the relation changes the graph. (E) Have students write relations and exchange these in the class for evaluating and graphing. (R) Give a table of values for students to construct a graph on grid paper.	Houghton Mifflin Mathematics 8, "Activities Courseware Solutions: One Equation", p. 67. Looking for a pattern is one problem-solving technique. Collecting and organizing information in charts and graphs is a problem-solving skill.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
8. Given a linear relation, constructs a table of values and a graph for that relation. (cont'd)	By graphing, for example $y = 2x + 3$, and $y = 2x + 5$ on the same axis, students explore how changing one part of the relation changes the graph.		

GRADE 9
MATHEMATICS

PLEASE NOTE

The following resources have received basic status and are available from the Learning Resources Distributing Centre:

Student texts for JOURNEYS IN MATH 7, 8
Student texts for HOLTMATH 7, 8, 9

Any other titles mentioned in this guide have not received formal status approval. They are presently under review and cannot, at this time, be obtained through the Learning Resources Distributing Centre.

CURRICULUM GUIDE (DRAFT)

PROBLEM SOLVING **Grade 9**

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Demonstrates an understanding of a problem-solving situation.	Refer to introductory comments under Objective #1 in Grade 7.		
2 Demonstrates a willingness to find a solution to a problem.	Refer to introductory comments under Objective #2 in Grade 7.		
3. Uses a variety of strategies to solve problems. Previously developed strategies are used. The following strategies should be developed throughout the various strands of the program and within the problem-solving framework: a) Understanding the problem • considers alternative interpretations • makes assumptions	As students encounter more complex problems, the skills required to solve them become more intellectually complex. Consequently, students must utilize higher-level thinking skills such as logic or reasoning. As a teacher demonstration involving the class, the strategy of using logic and reason can be developed within the problem-solving framework (Understanding the problem, Developing a plan, Carrying out the plan, Looking back) as follows: Margie is a blonde, Rose Mary a redhead, and Shirley is a brunette. They are married to Alex, Frank, and John but a) Shirley does not like John.	Teachers must recognize that problem-solving skills are essential for all students and that being perplexed, when first encountering a problem, is normal. Problems presented to students should be challenging yet solutions must be attainable to insure that students experience success. It is very important for teachers to realize individual student differences in learning; therefore the growth expectation should also vary. Students who experience difficulty with the complex strategies may find it necessary to use a more concrete approach for a longer period of time and may require more teacher guidance.	a) The use of calculators in problem solving must be encouraged so that time spent on tedious calculations is decreased and feedback on strategies is faster. Numbers from realistic and relevant situations are less imposing if calculators are used. b) Group work should often be used in problem solving. A student in a group deals with ideas and questions from other members of the group, and this may help each student to progress in developing problem-solving strategies.

Note: (E) = Enrichment
(R) = Remediation

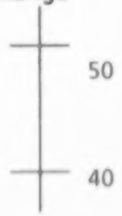
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3 b) Developing a plan (choosing a strategy) • formulates an equation • uses logic or reason • constructs flow charts • develops a symbol or code system • recognizes limits and eliminates possibilities c) Carrying out the plan • applies selected strategies • presents ideas clearly • documents the process • works with care • works in a group situation d) Looking back • generalizes solutions • creates and writes routine and non-routine problems	b) Rose Mary is married to John's brother. c) Alex is married to Rose Mary's sister. Who is married to whom? Assume that married people like each other! (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985 p. 40, #17.) Under the guidance of the teacher, students investigate the strategy by solving a similar but non-related problem such as: There are eight baseballs, all exactly alike in size and appearance, but one is heavier than any of the other seven which are all the same weight. With a balance scale, how can the heaviest baseball be positively determined with only two weighings? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 35, #9.9.) The natural progression leads to students using the same strategy to solve problems such as: a) A ferryboat, when filled, can carry 6 Pintos and 7 Toyotas or 8 Pintos and 4 Toyotas. If the ferryboat carries Toyotas only, then what is the maximum number that it can carry? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 42, #39.)	A teacher should challenge the more capable students by having them not only justify their strategies and solutions but also to consider other possibilities such as: • other strategies and solutions • "what if?" (change an element of the problem) • generalization of rules to other situations.	c) Computers may be used to assist in teaching problem solving. Various programs and simulations require the use of particular or various strategies. d) The use of relevant and realistic problems (from local sources such as newspapers and magazines) is encouraged because this will increase the interest of the students. Students may contribute their own ideas of problems; in addition to collecting data they may make up questions related to the information. An exchange of problems and questions may be encouraged. Examples can be obtained from such sources as: Mathematics Activity Software (MAC) by Houghton Mifflin, Minnesota Educational Computing Consortium (available through ACCESS) and Sunburst Communications Software.

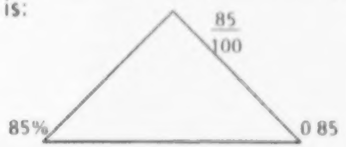
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
d) Looking back (cont'd) • generalizes solutions • creates and writes routine and non-routine problems.	b) Bill, John, Joe, and Henry have to catch the six o'clock bus. i) Bill's watch is 10 minutes fast, but he thinks it is 5 minutes slow. ii) John's watch is 10 minutes slow, but he thinks it is 10 minutes fast. iii) Joe's watch is 5 minutes slow, but he thinks it is 10 minutes fast. iv) Henry's watch is 5 minutes fast, but he is under the impression it is 10 minutes slow. If each leaves to catch the bus he will just make it, if his time is what he thinks it is. Who misses the bus? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 44, #5.) Evaluation To evaluate problem solving requires more than grading the solution to mathematical problems. Continual observation and questioning of students while they are solving problems is essential. • willingness to attempt problems • use of systematic approach • selection of appropriate strategies		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
d) Looking back (cont'd) • generalizes solutions • creates and writes routine and non-routine problems.	• efficiency in selection of appropriate strategies • logical justification of strategies and solutions • growth in confidence in problem-solving ability • transfer of problem-solving skills to situations other than mathematics. (Evaluation techniques and instruments for problem solving are found in the Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, pp. 7, 8, 52-56.)		

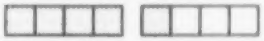
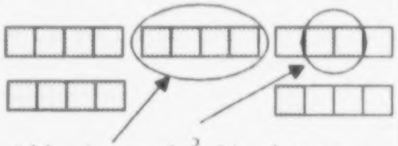
NUMBER SYSTEMS AND OPERATIONS

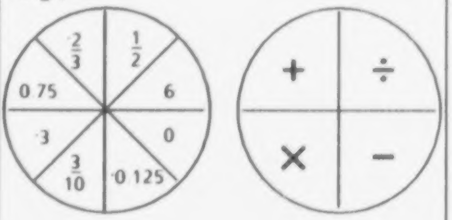
Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.			
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.	An equal emphasis should be placed on the various strategies for computing. Single-digit basic facts should be drilled on a regular basis through activities such as timed challenges or games. Paper-and-pencil strategies should be used to develop an understanding of sub-concepts such as re-grouping, borrowing or place value. Long and tiresome paper-and-pencil drill is discouraged. Estimation should be done on a daily basis. Recognition of appropriate situations for estimates, determining how precise an estimate should be for a given situation and knowing when a computed answer is possible are among skills to be emphasized. Mental computation involves using natural and easy strategies to compute exact answers. Strategies should be identified and shared as they evolve. Calculators should be used to develop understanding, to investigate patterns, and to perform tedious computations that do not enhance understanding.		A good calculator estimation activity is the "Range Game". Estimation and Mental Computation. Reston, VA: National Council of Teachers of Mathematics, 1986, pp. 182-185. "The Range Game" "The Range Game" builds estimation skills. The only tools needed are a calculator and paper and pencil for record keeping. The object is to find numbers within a given range that will satisfy a given equation. Introduce the game by displaying a range and a partial equation, as shown below. **Example:** Range 15 + _____ = _____ 

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
2. Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations. (cont'd)			Ask questions such as the following: • "Find a number that when added to 15 gives a sum in the range shown." • "What is the largest number that works?" "The smallest?" • "Are there any other numbers?" • "Let's find all the numbers that will work." List responses on the board and discuss the findings. Ask, "How many numbers work?"
3 Maintains previously developed skills with whole numbers, integers, decimals and fractions (operations, ordering, relationships among systems, need for rational numbers, order of operations).	Use time drills with basic number facts, in particular multiplication and addition. Encourage estimation, mental computation and the use of a calculator. e.g., An activity that demonstrates a relationship among number systems is: 	(R) Rather than doing the actual division ask students to write the number of digits in the whole number part of the quotient. e.g., 0.98) $\overline{10.03}$ 2 digits	Suggest calculator problems such as those from NCTM "Activities for Junior High School and Middle School Mathematics", p. 202. Play a game called "Erase". Have your friend enter any six-digit number into the calculator. Now see what is the fewest number of moves it takes you to get to a display of zero. For each move, you may add, subtract, multiply, or divide by any two-digit, non-zero number.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3 Maintains previously developed skills with whole numbers, integers, decimals and fractions (operations, ordering, relationships among systems, need for rational numbers, order of operations). (cont'd)	TRIPLE CHECK Usually by the first of March, my sixth-grade pupils have demonstrated that given a rational number in any of the three forms, fraction, decimal, or percent, they can quickly give the two equivalent names for commonly used fractions and can find equivalents for other fractions, decimals, or percents. To maintain and increase that knowledge, we use Triple Check at various intervals throughout the rest of the school year. On the last day of each month, pupils give their daily attendance, reading and mathematics grades, and spelling scores in "Triple Check". Usually we begin with a fraction, for example, for 17 days of attendance of a possible 20 days, we would write $\frac{17}{20} = \frac{85}{100} = 85\% = 0.85$. However, if you are working with decimals or percents, name them first. Triple Check encourages pupils to check their answers in a reasonable and an exact manner. It reinforces the idea that fractions, decimals, and percents communicate the same information about rational numbers. (From the file of Celestine Wyatt, 10629 S. Emerald, Chicago, IL 60628. Arithmetic Teacher.)	(R) Ask students to place decimal points in four different ways to make four true number sentences in each set. e.g., $124 \times 62 = 7688$ $124 \times 62 = 7688$ $124 \times 62 = 7688$ $124 \times 62 = 7688$ (Idea from Arithmetic Teacher, Vol. 34, #7, March 1987.) (E) Investigate the stock market, 'Why are fractions used rather than decimals?'	Trade places and have your friend try to "erase" a six-digit number you enter into the calculator. What did you notice about numbers that make them easy or difficult to erase? (From NCTM "Activities for Junior High and Middle School Mathematics", p. 202.)

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Performs the operations of addition, subtraction, multiplication and division with rational numbers.	Review the operations with positive rational numbers (see Grade 8 Objective #4) and the operations with integers (see Grade 8 Objective #3). Concrete activity: $-2\frac{3}{4} + 1\frac{1}{4}$ Use teacher-made manipulatives consisting of rectangular shapes divided into quarters. (Use overhead.) Blue (negative rational) Red (positive rational)  To find the answer to the above problem balance as many complete boxes and quarters of boxes as possible. e.g., 1 red box balances 1 blue box. $\frac{1}{4}$ red box balances $\frac{1}{4}$ blue box. Blue Red  This leaves 1 blue box and $\frac{2}{4}$ blue box or $-1\frac{2}{4} = -1\frac{1}{2}$ as the result.		Holtmath 7 Software Disks 3 and 5 Houghton Mifflin MAC 7

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Applies the rules for order of operations to evaluate expressions involving rational numbers in any of their forms.	Review the rules for order of operations with whole numbers, fractions and integers. Extend the concepts to rational numbers.	(R) Working in groups of two, develop spin game. Teacher instructs how many spins of each. A problem involving order of operations emerges. Have students estimate and then calculate their answers. A discussion will follow focusing on order of operations rules. e.g.,  -2/3 1/2 0.75 6 -3 0 3/10 -0.125 + ÷ × − Encourage the use of calculators.	Explore the operating system of a calculator by entering an operation in sequence. Ask the students to explain the operating system of their own calculator (e.g., Is the order of operation "built in"?).

NUMBER SYSTEMS AND OPERATIONS

Grade 9

Objective

6. Converts rational numbers from "$\frac{a}{b}$" form to decimal form (limit: b< 10 or b is a power of 10).

Clarification or Example

Students have converted rational numbers in the form "$\frac{a}{b}$" to decimal form by converting to denominators of 10, 100, or 1000 and by using a calculator.

Students should now be introduced to the paper-and-pencil algorithm of dividing the numerator by the denominator.

Stress should be placed on the meaning of the symbol "-" i.e., means 3 out of 4, three quarters and 3 divided by 4.

Emphasis should be placed on the following:

a) meaning of symbols ÷ and $\frac{a}{b}$ (bar)

b) numbers can have different forms (decimal, fraction).

Elective Suggestions

(E) Explore the resulting decimal forms of different rationals ($\frac{a}{b}$) - terminating, non-terminating, repeating or non-repeating. In particular an interesting activity is to contrast fractions with denominators of 11 to fractions with denominators of 9.

(E) Post this activity on the bulletin board but let students determine the answers.

D	100th digit of 1/D	D	100th digit of 1/D
2	0	11	9
3	3	12	3
4	0	13	9
5	0	14	4
6	6	15	6
7	8	16	0
8	0	17	8
9	1	18	5

(E) Explore non-terminating decimals (irrational numbers) e.g., "π" in Journeys in Math 9 TRM, p. 81.

Integration of Problem Solving and Technology

Calculate the digits in the decimal equivalents of common fractions using the following BASIC program for division.

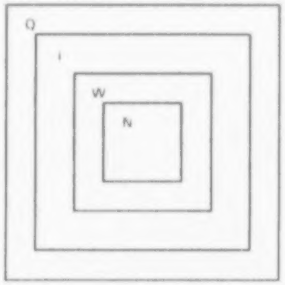
```
  5 REM Decimal equivalents
 10 HOME
 20 PRINT "This program will find
    decimal equivalents to 5 and 8
    decimal places."
 30 PRINT
 40 INPUT "What is the
    numerator?";D
 50 INPUT "What is the
    denominator?";D
 60 F = N / D
 70 F5 = INT(F*(10^5) + .5)/
    INT(10^5)
 80 F8 = INT(F*(10^8) + .5)/
    INT(10^8)
 90 PRINT
100 PRINT "The fraction,
    "N;"1";D;" is equal to:"
110 PRINT F5; "(to 5 places)"
120 PRINT F8; "(to 8 places)"
130 END
```

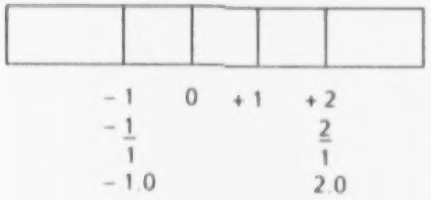
<table>
<tr><th></th><th>Clarification or Example</th><th>Elective Suggestions</th><th>Integration of Problem Solving and Technology</th></tr>
<tr><td>6. Converts rational numbers from $\frac{\text{"a"}}{\text{b}}$ form to decimal form (limit: b< 10 or b is a power of 10). (cont'd)</td><td></td><td>(E) Change repeating decimals to fractions.
See Holtmath 9 Teacher's Edition, pp. 76-77.
(R) 4-in-a-Line
Rules
• Take turns. Pick any two of these numbers to make an $\frac{a}{b}$ fraction.
3 4 5 6 8
• Divide. Mark the decimal name for your fraction on the game board (use x or o).
• Four marks in a line wins.
Game Board<table><tr><td>1.3</td><td>0.6</td><td>0.625</td><td>1.25</td></tr><tr><td>1.6</td><td>0.375</td><td>0.83</td><td>0.75</td></tr><tr><td>0.8</td><td>1.2</td><td>0.6</td><td>1.5</td></tr><tr><td>1.6</td><td>2.6</td><td>0.5</td><td>2.0</td></tr></table>NCTM – Feb. 1984</td><td></td></tr>
<tr><td>7. Converts rational numbers from decimal form to $\frac{\text{"a"}}{\text{b}}$ form (limit: terminating decimals).</td><td>(Holtmath 9 Teacher's Edition, pp. 76-77.)
Encourage mental computation and check answers using calculators.</td><td></td><td>Can use a calculator to explore patterns.
(See Journeys in Math 7 TRM, p. 191.)</td></tr>
</table>

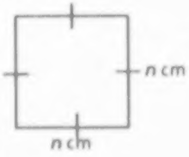
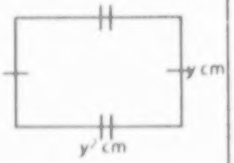
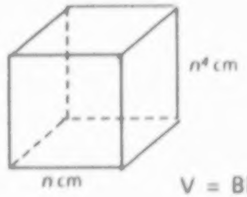
<table>
<tr><th>Objective</th><th>Clarification or Example</th><th>Elective Suggestions</th><th>Integration of Problem Solving and Technology</th></tr>
<tr>
<td>8 Computes the square root of whole numbers using estimation and a calculator.</td>
<td>Encourage students to estimate before calculating.

Start with the idea of squaring. 144 means 12 × 12. Then go into the opposite operation of square roots.
i.e., $8 \times 8 = 64$
$\therefore \sqrt{64}$ is 8

Establish a table to show squaring and square root are opposite operations.
e.g., ←→<table><tr><th>n</th><th>n^2</th></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>9</td></tr></table>
See Holtmath 9 Teacher's Edition, p. 174.</td>
<td>(R) Use prime factorization to find square roots.
$\sqrt{576} = 24$
$576 = \underbrace{(2 \times 2 \times 2 \times 3)}_{(24)} \underbrace{(2 \times 2 \times 2 \times 3)}_{(24)}$

2)576
2)288
2)144
2) 72
2) 36
2) 18
3) 9
3) 3
1
See Holtmath 9 Teacher's Edition, p. 176.</td>
<td>MAC 8, Disk B,
"Square Root Chase Game" reviews estimation, squares and square roots.</td>
</tr>
<tr>
<td>9. Demonstrates the relationship among whole numbers, integers and rational numbers.</td>
<td>The intent is for students to demonstrate the relationship between the sets. One way this can be demonstrated is pictorially.
 </td>
<td>Have students complete a chart.<table><tr><th></th><th>N</th><th>W</th><th>I</th><th>Q</th></tr><tr><td>1</td><td></td><td></td><td></td><td></td></tr><tr><td>$\sqrt{3}$</td><td></td><td></td><td></td><td></td></tr><tr><td>$\frac{3}{5}$</td><td></td><td></td><td></td><td></td></tr><tr><td>-0.125</td><td></td><td></td><td></td><td></td></tr><tr><td>π</td><td></td><td></td><td></td><td></td></tr><tr><td>3</td><td></td><td></td><td></td><td></td></tr></table></td>
<td></td>
</tr>
</table>

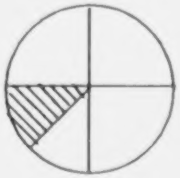
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
9. Demonstrates the relationship among whole numbers, integers and rational numbers. (cont'd)	Use a fraction tape to illustrate the relationship.  -1 0 $+1$ $+2$ $-\frac{1}{1}$ $\frac{2}{1}$ -1.0 2.0		
10. Understands and uses the following properties (limit: numerical bases): • $a^x \times a^y = a^{x+y}$ • $a^x \div a^y = a^{x-y}$ • $(a^x)y = a^{xy}$ • $a^1 = a$ • $a^0 = 1, a \neq 0$ • $a^{-x} = \frac{1}{a^x}$ (limit: a = 10).	For a concrete approach refer to the Grade 7 and 8 Number systems and Operations sections of this curriculum guide. Have students explore and generate exponent laws using numbers. e.g., 1. $5^2 \times 5^3 = 5^{2+3}$ 2. $5^3 = 125$ $5^1 = 5$ $5^0 = 1$ $5^{-1} = \frac{1}{5^1}$ $5^{-2} = \frac{1}{5^2}$ 3. $5^5 \div 5^3 = 5^{5-3}$ 4. $(5^2)^3 = 5^{(2)(3)}$		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
Extended Content Understands and uses the above properties from Objective #10 as literal bases.	Have students explore and generate exponent laws using literal bases. e.g., 1. $(x^2)(x^3) = (x)^{2+3}$ 2. $(x^{-2}) \div (x^{-3}) = \left(\frac{1}{x^2}\right)\left(\frac{1}{x^3}\right)$ $= \frac{1}{x^{2+3}}$ $= \frac{1}{x^5}$	(E) Find area and volume. e.g.,    V = Bh V = $(n^2)(n^4)$ V = n^6 cubic cm	
11 Writes large and small numbers in scientific notation: (e.g., $0.00008 = 8 \times 10^{-5}$).	Discuss the need to write very large or small numbers. (See Holtmath 9 Teacher's Edition, p. 170.) At the Grade 9 level, this could be extended to negative exponents. e.g., 0. 000 000 000 002 563 = 2.563×10^{-12}	(R) Multiply by powers of 10 (positive and negative exponents).	

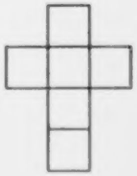
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
Extended Content Computes with numbers written in scientific notation (multiplying and dividing).	Apply exponent laws to computing numbers in scientific notation. e.g., $(8.7 \times 10^{12}) \times (3.3 \times 10^{-8})$ $= (8.7 \times 3.3) \times (10^{12} \times 10^{-8})$ $= 28.71 \times 10^{4}$ $= 2.871 \times 10^{5}$ Start with computing large numbers, i.e., 6 823 000 × 7 433 000. Is there a better way of handling this? Can lead to calculators and computers in handling scientific notation.	(E) Develop a method for adding or subtracting numbers in scientific notation. e.g., 1. $5 \times 10^{5} + 6 \times 10^{3}$ $= 5.06 \times 10^{5}$ 2. $8 \times 10^{8} + 5 \times 10^{1}$ $= 8.000\,000\,5 \times 10^{8}$	Using calculators and computers, multiply large numbers and observe the results when the instruments express the answers in scientific notation. i.e., 3.43501E + 05.

RATIO AND PROPORTION

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.			
2. Maintains previously developed skills (understands and constructs ratios, rates and proportions; finds the missing element of a proportion; writes ratios as percents; converts fractions and decimals to percents and percents to fraction and decimal forms; finds missing values in commission, sales tax, and discount situations).			Holtmath 7 Software Disk 4.
3. Converts fractional percents to fraction and decimal forms: e.g., $12\frac{1}{2}\% = \frac{1}{8} = 0.125$	Encourage the use of calculators to find commission, sales tax and discount. Review equivalent forms: ratios, percents, decimals (limit: whole no. %'s).	(R) Encourage mental computation with simple fractional percentages.	

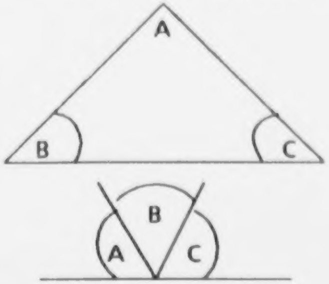
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Converts fractional percents to fraction and decimal forms: (cont'd) e.g., $12\frac{1}{2}\% = \frac{1}{8} = 0.125$	Develop the concept of fractional percentages at a semi-concrete level before formal development occurs. e.g.,  12.5% Use flash cards for drill and practice. Encourage the use of calculators.	Have students look for patterns: e.g., $100\% = \frac{8}{8}$ (1) $87\frac{1}{2}\% = \frac{7}{8}$ $75\% = \frac{6}{8}$ ($\frac{3}{4}$) $62\frac{1}{2}\% = \frac{5}{8}$ $50\% = \frac{4}{8}$ ($\frac{1}{2}$)	
4. Finds any one of the missing elements (value or percent) in applications such as simple interest, commission, sales tax, discount, profit and loss, and percent increase and decrease situations.	This objective is an extension of Objective #5 (Ratio and Proportion, Grade 8) where students were always given the percent and asked to find a missing element. Finding the percent (given other elements) is subtly different. Solutions will occur in fraction or decimal form and must be converted to percent.	(E) Have students determine monthly payments for a car loan. Provide students with information as to: down payment required; current interest rate; length of payment period.	Problem-Solving Project Investigate the cost of purchasing a car vs renting a car. e.g., payments interest insurance maintenance gas

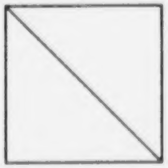
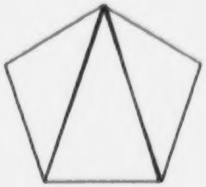
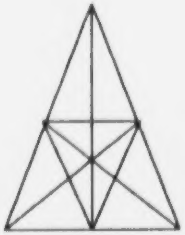
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4 Finds any one of the missing elements (value or percent) in applications such as simple interest, commission, sales tax, discount, profit and loss, and percent increase and decrease situations (cont'd)		(E) Have students investigate the difference in total amount of money payable if a house mortgage is amortized over 15 years, 25 years and 35 years. Amortization tables are required. (E) With the use of calculators, have students calculate and compare the amount of interest they would earn if a given sum of money was placed in a bank account where interest was compounded semi-annually or monthly. Discuss different bank accounts and the advantage of one over another. i.e., DICA Regular savings (interest twice a year) Daily interest.	Compound interest is not always calculated annually. It could be calculated semi-annually, quarterly, monthly, or daily. Find a formula to calculate interest compounded "n" times during a year (From Holtmath 9 Teacher's Edition, p. 209.)
5 Interprets maps and scale drawings.	Use a class set of atlases to generate a discussion about scale: a) What is it? b) How is it expressed? c) Calculate actual and scale distances. Discuss trips students have taken and use maps to get the actual from the scale distance.	(E) Students are planning a trip in which they rent a car. They want to calculate costs for distance using the scale drawing of a map. (R) Review of metric conversions cm to km, cm to m.	

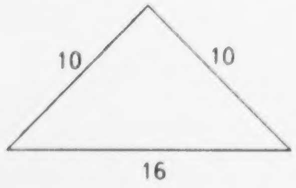
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
6. Uses a scale to construct drawings, maps or pictures.	Ask students to draw a cartoon character, a company logo, a rock band logo, a map or a plan of a building, room, etc., on a smaller (or larger) scale. Students should be taught how to enlarge or reduce with a scale. The criteria for evaluating the project should be clearly defined (e.g., accuracy of scale, proportion, neatness).	Students who experience difficulty with this should be encouraged to use grid paper to help them with proportion. (E) Give students two drawings, actual and scale. Have them work out the key. (E) Have the students measure and draw a scale diagram of the classroom.	Have a student use scale drawings in problems such as: Merv has a garden plot 12 cm by 9 m. He wishes to plant vegetable plots 2 m by 9 m. What is the maximum number of plots Merv can plant? How many arrangements are possible?
7. Applies ratio and proportion in practical situations (e.g., uses shadows to find the height of a pole or building; comparative shopping; building a model, computing a test or report card mark based on weighted averages).	Teachers direct their students to develop a project in which they build an actual model from a scale. e.g., school logo cars mascot e.g.,  From this 'net' drawing construct a model which is 6 times as large (1:6).	(E) Give students their marks for the term with each of the assigned weightings. Have them calculate their term marks. Compare the results to the computer printout. (R) Have students check unit prices in their local grocery store. (R) Determine which item is the better buy by finding the unit prices. e.g., 3 blank tapes for $5.99 or 2 blank tapes for $4.69.	Have student investigate problems to determine when the most economical buy is not always the best buy.

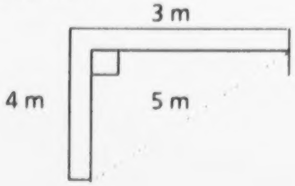
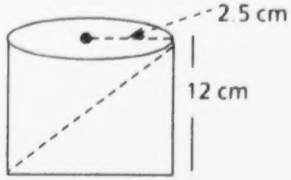
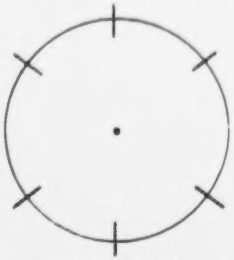
297

298

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.			
2. Maintains previously developed skills (linear, area, volume, capacity, and mass units of measure; classification of polygons; perimeter and area of polygons and the circle; volume of a right rectangular prism and cube).			Problem: You have a 5 L jar and a 3 L jar, both of which are not marked in any way. You may use as much water as you need. Describe how to obtain exactly 4 L of water by filling and emptying the jars.
3. Uses concrete manipulatives to determine the sum of the angles in a triangle (180°).	Draw and cut out any triangle. Then cut or tear each angle from the triangle. Place them together to show that the sum of the interior angles equals 180°. 	Remediation: Have students measure and record angles of various triangles, then determine the sum for each. This exercise is an appropriate point to discuss accuracy of measurements.	

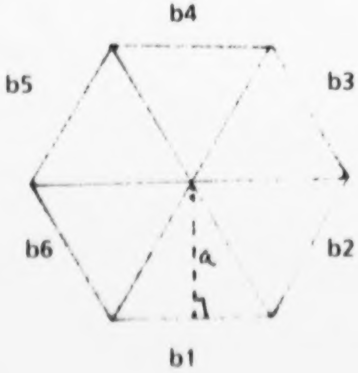
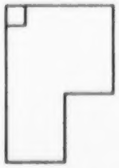
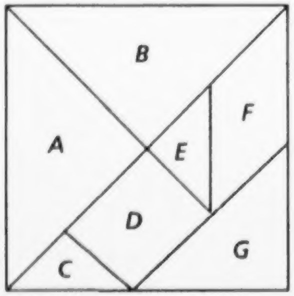
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Determines the sum of the interior angles in polygons.	Explore various strategies that could be used to determine the sum of angles in any given polygon. One possibility is by measuring each angle with a protractor and then determining the sum. Another strategy is to divide the shape into triangles as shown. e.g.,  square = 2 triangles = 2 x 180° = 360° ∴ sum of interior angles is 360° e.g.,  pentagon = 3 triangles = 3 x----- = --------- ∴sum of interior angles is -----		Problem- Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 47. Problem: How many triangles can you count? 

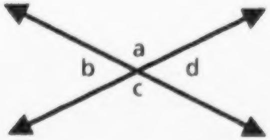
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Uses concrete manipulatives to develop the Pythagorean relationship in right triangles.	(See Holtmath 9 Teacher's Edition, p. 179, "Alternative Teaching Strategies".)	(E) Explore Pythagorean triplets. A possible approach is to have students write a computer program to generate Pythagorean triplets. Use a calculator to generate Pythagorean triplets using the expressions: $(m^2 - n^2, 2mn, m^2 + n^2)$ (E) Explore the contention that the products of Pythagorean triplets are always divisible by 60. Does it work? Why or why not?	
6. Applies the Pythagorean relationship to practical situations.	The Pythagorean theorem lends itself to solving many practical problems. Some strategies to reinforce are drawing a diagram, making a simple problem, developing a code or symbol system, documenting the process, looking back to determine if the solution is reasonable, making and solving other similar problems.	(E) The Pythagorean theorem can be used to find the distance between two points on a grid. (E) Research the historical development of the Pythagorean theorem.	Problem a) An isosceles triangle with sides of length 10 cm and 16 cm is folded in half. How long is the crease? 

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
6. Applies the Pythagorean relationship to practical situations. (cont'd)	e.g., A carpenter wishes to build a square corner in framing a basement. He measures distances of 3 m and 4 m on the 2 x 4's on the floor. The distance of 5 m ensures a square corner.  		b) Find the length of the longest needle that can be placed in a cylinder that has a radius of 2.5 cm and a height of 12 cm. 
7. Constructs regular polygons using tools such as a computer, ruler, protractor and/or compass.	After discussing regular polygons, work through a construction. e.g., an inscribed hexagon.  – Why does it work? – How can we use this to get an equivalent triangle? – How can we get an inscribed square? Octagon?	(E) Have students create designs using polygons constructed.	Use a graphics or LOGO program to generate regular polygons. Investigate their attributes: (a) measure of angles (b) measure of sides (c) relationship of perimeter to diagonal (diameter)

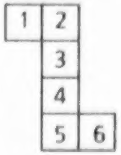
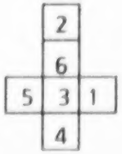
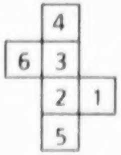
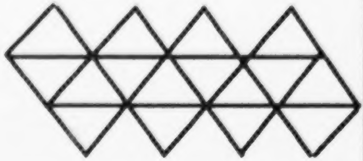
MEASUREMENT AND GEOMETRY

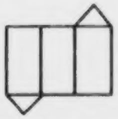
Grade 9

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
8. Understands and uses a strategy to determine the area of a regular polygon.	Build on previous skills (area of triangles, parallelograms, and trapezoids). Expand strategies to find areas of many polygons. For example, a hexagon can be divided into 6 triangles.  Total area = sum of 6 triangles. A = $\frac{1}{2}a(b_1 + b_2 + \ldots + b_6)$ A = $\frac{1}{2}ap$ A = $\frac{1}{2}$ (apothem) (perimeter) Note: p = ns (perimeter = no. of sides + length of side)	(E) Expand basic strategies to finding areas of compound figures. e.g.,  The area of this can be found by dividing the figure. (R) Use cutout pieces to make shapes. (Arithmetic Teacher, Vol. 31, #5, January 1984, pp. 28-32.)	*The figure shows a square separated into the seven pieces of an ancient puzzle called the **tangram**. If the area of the entire square is one square unit, what is the area of each of the seven tangram pieces?  *Creative Problem Solving in School Mathematics, Houghton Mifflin, p. 134.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
9. Identifies pairs of angles: (supplementary, complementary, adjacent and opposite).	This is the appropriate time to reinforce the use of a ruler and a protractor. Have students draw two intersecting lines and then measure the resulting angles. They will then discover the resulting relationships.  Angles: a + b = 180° b + c = 180° c + d = 180° d + a = 180° a = c b = d		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
10. Uses a compass and a straightedge to construct: • a congruent segment • a congruent angle • a perpendicular bisector of a segment • a bisector of an angle • a perpendicular to a line • angles of 90°, 45°, 60°, and 30°.	(Holtmath 9 Teacher's Edition, pp. 220-229.) Angle construction is a direct extension of the following angles: 90°– a perpendicular 45°– bisector of a 90° angle 60°– construction of an equilateral triangle 30°– bisector of a 60°angle	(E) Explore other constructions, such as constructing a perpendicular to a point not on a line, finding an altitude of a triangle.	
11. Given nets, constructs right prisms.	Given a net of a right rectangular prism, have students fold and paste to make a box. Explore its construction making note of its characteristics (faces, edges, vertices). Expand this to students exploring other right prisms (for example, a cube, a triangle prism, a pentagonal prism) and their construction.	(E) Show students examples of other figures (tetrahedron, dodecahedron, and so on). Explore the nets for these. (R) Have a collection of boxes (cracker boxes, cereal boxes, matchboxes, etc.). Students can then dismantle the boxes and examine the nets. Encourage them to look for the similarities and differences.	Problem Solving: Discuss how to produce nets of right prisms or cylinders, given specific conditions. e.g., You have a piece of paper (looseleaf size). Construct a net for a cylinder that has a 5 cm radius. Problems of this type have more than one solution since the condition net specified is the height.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
11. Given nets, constructs right prisms. (cont'd) Extended Content Given nets, constructs right pyramids.		Problem:    Each of the above patterns can be folded to form a cube. Which two cubes will look the same? (E) String art on an octahedron constructed of drinking straws. (Arithmetic Teacher, Vol. 34, #3, November 1986, pp. 30-33.)	*Have a three-dimensional model of an icosahedron available for students to examine. Give everyone a pattern for constructing an icosahedron. Ask each student to colour a pattern such that, after the figure has been cut out and taped together, no two adjacent faces (faces that share an edge) will be the same colour. Furthermore, everyone should attempt to use the fewest number of colours. Finally, have the students cut out their patterns and tape them together to check their solutions. (diagram)  *(Arithmetic Teacher, Vol. 31, #7, March 1984, p. 43.)

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
12. Classifies right prisms and cylinders.	From exploration of constructing nets, students will have an intuition as to the characteristics of various right prisms and cylinders. Classification is simply a natural progression. i.e., rectangular right prism triangular right prism square right prism (cube) right circular cylinder		
Extended Content Classifies right pyramids.	Have models of various pyramids and discuss common attributes and differences. Use words such as faces, bases, edges and vertices.		
13. Understands and uses a strategy for finding the surface area of any right prism or cylinder.	The development of a strategy for finding the surface area of right prisms and cylinders can be easily approached by building on students' investigation of nets. Some students may divide prisms into faces, while others may decide to develop a formula. i.e.,  A triangular prism is made up of 3 rectangles and 2 triangles.	(R) Using models of prisms and cylinders, take them apart and explore the area of each piece.	Problem Solving: Strategies can be developed and expanded upon through problem solving. For example "Susan built a toy chest for her little brother. The dimensions of the toy chest are 95 cm by 40 cm by 50 cm. Susan wanted to paint the outside (including the bottom) with enamel paint. What is the total surface to be painted?"

315

316

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
14. Understands and uses a strategy for finding the volume of any right prism or cylinder.	Students, in previous work, have used a formula for finding the volume of a cube or right rectangular prism. Using blocks, build a right rectangular prism. By dividing the prism into two congruent triangular prisms, a strategy for determining the volume of a triangular prism should be apparent.	(E) Write a computer program that can be used to calculate the volume of right prisms and cylinders. (E) Explore how changing one characteristic of a prism or cylinder changes its volume. For example: A cylinder with a height of 6 cm and a radius of 5 cm is changed to a cylinder with a radius of 10 cm. What happens to the volume?	Problem Solving – Apply strategies to solving non-routine problems. e.g., A solid cube is painted red on all sides. The cube is then cut into 27 equal smaller cubes. How many of the smaller cubes have red paint on only 2 sides? (Problem-Solving Challenge for Mathematics. Edmonton: Alberta Education, 1985, p. 47.)

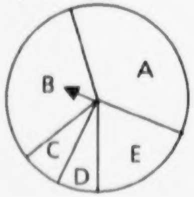
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
Whenever possible, the objectives of this strand may be integrated throughout the program. The effect is to make the objectives more meaningful than when taught in isolation.			
1. Applies and practises problem-solving skills in new situations.			
2. Maintains previously developed skills (understands purpose, use and misuse of statistics; biases in surveys; represents data in the form of pictographs, bar graphs, line graphs, circle graphs).	See Grade 8 Data Management, Objectives #2, 4.	(E) Watch a newscast on TV to determine when, where, how and why statistics are used. Discuss their accuracy and timeliness.	Problem-solving strategies and skills can be a natural outgrowth of a class survey. e.g., collect, organize and interpret data; make decisions; estimate; predict; draw inferences.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Analyzes and interprets arguments or conclusions based on statistical information.	Have students research statistical information from magazines and newspapers. Discuss 1. Is the result typical of the population? 2. Is the survey large enough to make prediction? 3. Are there any noticeable scale variations (e.g., bar graphs without a zero value)? 4. Is there any misrepresented data, incomplete data, or misleading data? (See Holtmath 9 Teacher's Edition, pp. 348-351.)		
4. In data from meaningful situations (e.g., test marks), understands and uses the terms mean, median, mode and range.	In using quiz scores (such as 18, 28, 27, 19, 24, 40, 27, 35, 27, 39, 28, 40, 25, 36, 33, 27, 27) determine the measures of central tendency: a) mean (average) b) range of values c) median (middle value) d) mode (most frequent value). Discuss when and why each of these measures may be used.	(E) Compare the measures of central tendency (mean, mode, median, range). Determine which is most appropriate within its set of data.	Houghton Mifflin MAC 8, Three M's. Students can design a simple computer program to calculate the mean and possibly the mode, median and range.

321

322

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. In data from meaningful situations (e.g., test marks), understands and uses the terms mean, median, mode and range. (cont'd)		(E) Find the effect on mean, median, mode and range if: a) a very high score is added b) the frequency of the lowest value is increased several times c) more than one value is most frequent.	
5. Distinguishes between a percent and a percentile.	The intent is to convey the meaning of percent and percentile in real-life situations. Define percent as being a fraction out of 100 and percentile as a ranking (from 1 to 99). Use examples to show the difference. e.g., When born, a baby had a mass of 5 kg. Later, the health nurse said that the baby weighed 8 kg (an increase of 60% in mass) and was in the lower 20th percentile of mass for babies of that age. Compare percents from test scores to the percentiles from the same tests. Students are not expected to calculate the percentiles but should understand them as rankings.	(R) Develop a chart based on test scores. Predict what percentile various scores would have. (E) Conduct a survey on height or weight of people and construct a table. Use the table to determine in what percentile people who were not surveyed would be.	

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
6. Conducts a survey or poll using correct sampling techniques and reports results using an appropriate table, chart and/or graph.	Decide on what would be a typical sample group. Conduct a survey and choose the best form for displaying the results. Did you use an unlimited response, such as, what is your favourite sweater colour?	(R) Use a restricted choice survey (e.g., What is your favourite dessert – strawberry jello; banana, butterscotch pudding; chocolate ice cream?) and decide on the target group. (E) Conduct a glasses survey: Who wears eyeglasses, contacts, sunglasses, combinations, and who doesn't? How would one answer the question of who wears glasses and misuse the information? Was the survey based on age groups, income groups, etc?	
7. Understands and uses the term probability.	Introduce the idea of the impossible and the certain event. e.g., A person can hold his breath for 1 hour. The sun will rise tomorrow morning. Explore a more systematic approach to probability.	(E) Have students examine lotteries and determine the chances of winning.	Students design a spinner that has 5 possible outcomes but the sectors are not equal. 

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
7. Understands and uses the term probability. (cont'd)	Probability can be defined as the chance of an event occuring. This can be calculated by writing a ratio of "the number of outcomes in the event, divided by the total number of possible outcomes". This can be discussed in terms of the probability of a coin toss coming up heads or of a dice roll to produce a certain number.		Students then estimate the probabilities of each outcome. Then they take several spins to see how close they came to the actual probability. From Journeys in Math 7 TRM.
8. Expresses the probability of the occurrence of an event from a practical situation or a simple experiment or simulation (e.g., pulling a particular coloured marble out of a bag or socks out of a drawer).	The student will predict probabilities and then conduct an experiment such as a coin toss to determine an outcome. Have each student toss a coin 10 times. Record and analyze results. Then, compile the class result and analyze them. Compare the two outcomes and discuss the validity and how close the results are to the predicted probability. Other examples may include rolling a dice, drawing a card from a deck of cards, or pulling a coloured marble from a bag of marbles.	(E) Students will predict probabilities and then conduct an experiment repeatedly rolling 2 dice. Record the results and determine how close they are to the predicted probabilities. (E) Predict the probability of drawing two aces in a row (or getting consecutive 7's with two dice) and then conduct an experiment to test the predictions. Predict probability with non-standard dice.	Houghton Mifflin MAC 8 Software Probable Urnings. Computer programs that will simulate coin tosses are available. These programs may be used instead of actual experiments. Write a BASIC program to generate random numbers from one to six and then tabulate how many times each number appears.

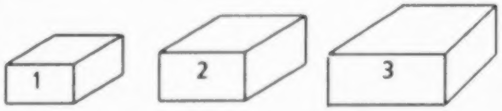
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
8. Expresses the probability of the occurrence of an event from a practical situation or a simple experiment or simulation (e.g., pulling a particular coloured marble out of a bag or socks out of a drawer). (cont'd)			e.g.,

```
  5  REM Random Numbers for the
     Apple
 10  DIM N(6)
 20  HOME
 30  INPUT "How many numbers do
     you want?"; X
 40  FOR Y = 1 TO X
 50  A = INT(RND(1)*6 + 1)
 60  FOR Z = 1 TO 6
 70  IF A = Z THEN N(Z) = N(Z) +
     1: GOTO 90
 80  NEXT Z
 90  NEXT Y
100  PRINT "Final Results"
110  FOR Y = 1 TO 6
120  PRINT Y, N(Y)
130  NEXT
140  END
```

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Applies and practises problem-solving skills in new situations.	All the ideas and steps involved in general problem solving should be applied to "real life" situations when possible.		
2. Maintains previously developed skills (variables; like terms, evaluation of expressions; solving equations; generating and plotting ordered pairs from a given relation).			Simple graphing programs can be used to generate ordered pairs and plot these pairs on the screen.
3. Uses formal procedures to solve equations (using all forms of rationals) of the form: $x + a = b$, $ax = b$, $ax + b = c$, and $\frac{x}{a} = \frac{b}{c}$ $ax + b = cx$, $a(x + b) = c$ and $ax + b = cx + d$	In addition to forms solved in Grade 8, students expand skills to solve three more forms. Equations may include all rationals. In the form $a(x + b) = c$, use of distributive property needs to be emphasized. For all forms, use addition, subtraction, multiplication or division of like quantities on both sides of the equality to form equivalent sentences.		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
4. Verifies solutions to equations.	It is assumed that students can solve equations formally. They should be encouraged to substitute the answer in the original equation and decide if the answer is correct (see Grade 8 Algebra, Objective #5).	(R) Use manipulatives such as a balance of scale to reinforce the concept of equality.	Use of a calculator is encouraged.
5. Manipulates a given formula to change the subject of the formula: e.g., given $x = \frac{f}{w}$ then $w = \frac{f}{x}$	The few formulas presented in mathematics classes are usually taught as a means to an end (e.g., $d = rt$ is used to find distance, rate or time). The result is that little transfer occurs to knowing how to use other formulas in mathematics and other disciplines. A much more holistic view of formulas must be taken. In Grade 8, the outcome of objective #6 (Algebra) is that students will understand relationships among the variables, recognize the similarities (or differences) among those relationships (regardless of what kind of variables are used) and will know how to substitute and solve for missing elements. The intent of this objective is to provide students with an alternative strategy for finding the missing value in a formula, especially in situations for which the required missing element is always the same.		Computers and calculators may be used to check some of the values on the chart.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
5. Manipulates a given formula to change the subject of the formula: (cont'd) e.g., given $x = \frac{l}{w}$ then $w = \frac{l}{x}$	For example, if students were required to find the principals in a number of loans, manipulating the formula first (using $p = i/rt$) is much more convenient than using the form $i = prt$ (and then having to substitute and isolate the p). Review the relationships among the variables in any formula (real or contrived) and demonstrate that the techniques for isolating a particular variable are the same whether the variable has been replaced with a number or not.	(E) Discuss the effect on a selected element of a formula if another element is doubled, tripled, etc. e.g., What happens to A, in $A = \pi r^2$, if r is doubled? tripled?	

<table>
<tr><th>Objective</th><th>Clarification or Example</th><th>Elective Suggestions</th><th>Integration of Problem Solving and Technology</th></tr>
<tr><td>5. Manipulates a given formula to change the subject of the formula: (cont'd)
e.g., given $x = \frac{f}{w}$
then $w = \frac{f}{x}$</td><td>A formula which can be practically verified is $D = \frac{M}{V}$
where D means Density
M means Mass
V means Volume
Have the students weigh and measure the volume of wooden blocks of different sizes (but same type of wood).


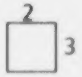
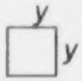
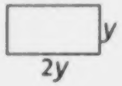
<table>
<tr><th></th><th>M</th><th>V</th><th>$D = \frac{M}{V}$</th></tr>
<tr><td>1</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td></tr>
<tr><td>4</td><td></td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td><td></td></tr>
</table>
When the chart is filled, verify the values using $M = DV$ or $D = \frac{M}{V}$</td><td></td><td></td></tr>
</table>

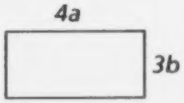
Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
6. Finds a missing element of a formula through manipulation.	This objective is an extension of the previous objective (#5). The development of objective #5 should be extended to include actual computations of values.		
7. Solves inequalities of the form $x + a \geq b$ and $cx \leq d$ (c is positive; direction of inequalities may vary).	(See Holtmath 9 Teacher's Edition, p. 143.)		Problems in which a maximum or minimum value may be explored.
8. Verifies solutions to inequalities.	To verify inequalities, it is necessary to determine the limits on boundaries. Substitute the maximum or minimum value. e.g., $3x>9$ $x>3$ Use $x = 3$ L.S. $= 3(3)$ $= 9$ R.S. $= 9$ L.S. $=$ R.S. The limits have been found to be correct.		

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
9. Graphs solutions to inequalities on a number line.	Reviews the meaning of the inequality symbols < and > and the combined symbols of ≤ and ≥. A method for plotting or a number line should be decided upon; i.e., "is more than" is the same as > and can use a circle (○——►) whereas "is at least" is the same as ≥ and can use a dot. (◄——●) (See Holtmath 9 Teacher's Edition, pp. 143-145.)	Draw graphs in which the values of x and y are between certain limits (domain and range respectively). Have the students graph the domain and range on a number line.	
10. Given a set of ordered pairs or a table of values, writes the function that determines the relation (limit: linear relations).	From a table of values students try to determine the relation by looking at the pattern or using guess and check. Relations should be limited to the form $y = ax + b$. (Plotting ordered pairs is a maintenance skill but may be reviewed here).	Have students make up their own tables from a relation. Exchange tables for others to determine the relation form. (E) Explore how changing *a* or *b* in a linear relation of the form $y = ax + b$, changes the graph. This can be extended to students making predictions concerning the direction of the line and the steepness of the line as *a* or *b* changes.	Houghton Mifflin, Mathematics Activities Software, Grade 8, pp. 66-68. Explore problems in which patterns develop, using logic or reasoning, generalities solutions. (E) Solve simultaneous linear relations by graphing and determining the point of intersection.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
1. Knows the terms associated with polynomials: monomial, binomial, trinomial, degree, numerical and literal coefficient.	(Holtmath 9 Teacher's Edition, pp. 256-257.)		
2. Find the sum and difference of monomials.	Use manipulatives in the form of squares and rectangles to develop the concept of "like" terms. Use □ and ▭ to represent the squares and rectangles. ▭ and ▭ are like terms. Therefore, ▭ + ▭ + ▭ + ▭ = 4 ▭ Finally use $3x$ and $4x$ as examples of like terms and $3x$ and $5y$ as unlike terms. $3x + 4x = 7x$. Conclude that to add or subtract like terms one must add or subtract the numerical coefficients. Remember to subtract, add the opposite. $4x - (^-2x)$ $= 4x + 2x$ $= 6x$		Computer: Sunburst Communications Polynomials Practice Using Tiles.

ALGEBRA-EXTENDED CONTENT **Grade 9**

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Finds the product and quotient of monomials.	(MULTIPLYING MONOMIALS) Give some numerical examples of products of the same factors, e.g., $4 \times 4 = 4^2$ $4 \times 4 \times 4 = 4^3$ $4 \times 4 \times 4 \times 4 = 4^4$ Use the same type of examples with letters instead of numbers $y \times y = y^2$ $y \times y \times y = y^3$ $y \times y \times y \times y = y^4$ Introduce the idea that the area of a rectangle is the product of two factors (or sides) e.g., $(2) \times (3) = 6$ square units   $(y)(y) = y^2$  $(2y)(y) = 2y^2$		Use the Diskette: Sunburst Communications – Polynomial Practice Using Tiles.

Objective	Clarification or Example	Elective Suggestions	Integration of Problem Solving and Technology
3. Finds the product and quotient of monomials. (cont'd)	Manipulatives to be used for the area concept may be cardboard cut out in rectangular shapes. After this introduce abstract examples. e.g., a. 5×4 b. $2x \times 3$ c. $(4x) \times (3x)$ Using a rectangle, note the calculation of area using sides of $4a$ and $3b$. To multiply two or more monomials, multiply the numerical coefficients and multiply the literal coefficients.  e.g., $(4a)(3b) = 12ab$ Using examples with numbers (i.e, $2^5 \div 2^2$) give the rule for division of powers with the same base. To divide monomials you divide the numerical coefficients and subtract the exponents on the same bases. i.e., $2^5 \div 2^2 = 2^{5-2} = 2^3$ $x^5 \div x^2 = x^{5-2} = x^3$		